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LOSS AND LINE FLOW ALLOCATION
IN A
COMPETITIVE ENVIRONMENT

by

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A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements for the degree of Master of Engineering

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ABSTRACT

Under open access, market driven transactions have become the new independent decision variables defining the behaviour of electric power systems. Understanding the impact of these bilateral transactions on system losses and line loading is important for the secure operation of the network and for establishing equitable tariffs corresponding to actual network use. The theory presented here is based on the argument that it is always possible to compute the exact loss and line loading allocation corresponding to an infinitesimal variation in a bilateral transaction. This leads to a set of governing differential equations whose solution yields the exact allocation. Numerical examples illustrate the properties of both the exact and approximate solutions of the allocation equations, as well as their dependence on the path of integration. The exact solutions are also compared with those obtained using pro-rata methods, DC power flow formulations, and contract paths. Lastly, practical applications of the allocation algorithm are suggested and discussed.

RÉSUMÉ

Sous le marché à accès ouvert, les transactions sont devenues les nouvelles variables de décisions indépendantes définissant le comportement des réseaux électriques. Comprendre l'impact des ces transactions bilatérales sur les pertes du système ainsi que sur la charge de la ligne est importante pour exploiter le réseau en sécurité et pour établir des tarifs équitables correspondant à l'utilisation réelle du réseau.

La théorie présentée ici est basée sur le fait qu'il est toujours possible de calculer l'allocation de la perte exacte , ainsi que la charge de la ligne correspondant à une variation infinitésimale de la transaction bilatérale. Cette théorie conduit à un ensemble d'équations différentielles dont la solution donne une allocation exacte. Des exemples numériques illustrent les propriétés des solutions exactes et approximatives des équations de l'allocation, aussi bien que, leur dépendance sur le chemin d'intégration. Les solutions exactes sont aussi comparées à celles obtenues utilisant les méthodes heuristiques, les formulations de modèle DC d'écoulement de puissance, ainsi que les chemins "contrat". Finalement, des applications pratiques de l'algorithme d'allocation sont suggérées et discutées.

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LIST OF SYMBOLS

α	Basis vector for the null space of the jacobian $\frac{dP}{d\delta}$
B	Susceptance matrix
β	Vector of flow sensitivities
δ	Vector of voltage phase angles
e	Vector of ones
ED	Matrix of real power transactions between trading entities and loads
EE	Matrix of real power transactions between trading entities
GD	Matrix of real power transactions between generators and loads
GE	Matrix of real power transactions between generators and trading entities
H_f	Matrix of contractual flow sensitivities
H_L	Matrix of contractual loss sensitivities
ITL	Incremental Transmission Loss coefficients
L	Matrix of contractual loss components
$P(\delta)$	Vector function relating net bus injections and voltage phase angles
P_g	Vector of real power bus generations
P_d	Vector of real power bus loads
P_{flow}	Arbitrary net real transmission line flow
P_f	Matrix of contractual line flow components
P_{loss}	Total real power losses
U	Loss index matrix
OASIS	On-line At the Same time Information System

Chapter 1

An Introduction and Review of Competition In the Power Industry

1.1 Introduction

1.1.1 Brief History of competition

Recently, regulated or centralized industries, telecommunications being a prime example, have moved towards increased competition. The power industry is the last major monopoly to undergo such change. This current trend towards deregulation, or decentralization, at all levels of the power industry is aimed at providing lower prices and more choices to electric energy consumers.[1,12]

In North America, the trend towards deregulation is an ongoing process which began in the United States in 1978 with the enactment of the Public Utility Regulatory Policy Act (PURPA), which introduced competition in generation. The introduction of PURPA forced utilities in the U.S. to purchase power from cogenerators and independent power producers (IPPs) at prices which equalled their "avoided cost". The introduction of competition in generation has proven to be a success as, by 1993, approximately fifty percent of the new capacity was attributed to IPPs.[1] The next major event was the Energy Policy Act (EPAct) of 1992, which provided wholesale customers a choice of suppliers and obliged transmission owning utilities to provide access to their transmission networks to electricity wholesalers. In 1995, the U.S. Federal Energy Regulatory Commission (FERC) issued two notices of proposed rule-making (NOPRs), and finally, on April 24, 1996, FERC issued Final Rules 888 and 889, addressing the issues of wholesale competition through non-discriminatory transmission services. The eventual goal of deregulation is to improve efficiency by promoting competition at all levels, including the retail level, as stated in the California Public Utilities Commission proposal of April 1994, which proposed that electric utilities be required

to provide retail wheeling to all customers by the year 2002.

The introduction of competition at all levels of the power industry, and the resulting decentralization, require a reorganization and rethinking of the traditional issues of power balance, security, and economy. Whereas these issues were ultimately the responsibility of one entity, these functions are now to a large extent shared. While there is still an Independent System Operator (ISO) with the authority to intervene whenever security is threatened, the issues of reactive power supply and voltage control, regulation and frequency response, energy imbalance, operating reserves, real power loss replacement, and transmission, rather than being the responsibility of the utility, each represent competitive environments within the power industry.

The present thesis addresses the issues of ancillary services, in particular loss allocation, as well as transmission line-loading allocation. The goal of this work is to, first and foremost, provide a theoretical foundation for loss and line-loading allocation, and secondly, to provide an overview of the topic of ancillary services and their role in power system operation, both in a centralized and decentralized environment. To this end, the thesis proceeds as follows:

Chapter 1: Overview of operational procedures, literature review, and statement of thesis.

Chapter 2: Theoretical development of loss and line-loading allocation equations.

Chapter 3: In depth discussion and investigation of the loss allocation problem.

Chapter 4: Detailed analysis of the flow allocation including its use in congestion management.

Chapter 5: The presentation of the loss and line-loading allocation for a 30 bus power system.

Chapter 6: Summary and Conclusions.

1.1.2 Claim of Originality

To the author's knowledge, this thesis represents the first work which allocates real power loss and transmission line loading to individual bilateral transactions between generators and loads. Unlike previous attempts, the developed theory is mathematically and physically sound, being based on exact network relationships. No assumptions or approximations have been made in the theoretical derivations.

1.2 Traditional Power System Operation

1.2.1 Introduction

The main issues involved in power system operation are power balance, security and reliability, and economy. [12] In the traditional vertically integrated environment, these issues are ultimately the responsibility of one entity, that is, the electric utility. Each issue is briefly addressed to convey an understanding of the key issues involved in traditional operational procedures.

1.2.2 Power Balance

Power balance must always be maintained; that is, the generation supply must satisfy the load demand. Traditionally, power balance is performed by the power system operator (PSO) in three basic stages: resource acquisition, that is choosing the types of resources required to meet forecasted load demand, often years in advance, scheduling, and dispatch, which includes economic dispatch and automatic generation control, the latter being responsible for maintaining real-time power balance.

In the traditional, vertically integrated utility, transmission losses are considered part of the total generation supply required to satisfy consumer demand. Since all generators belong to the same company it is unnecessary to determine the exact distribution of losses among the generators, or more specifically among the individual power transactions between suppliers

and loads as may exist in a deregulated environment.[12]

1.2.3 Security and Reliability

Security and reliability concern the ability of the power system to maintain continuous customer service in spite of generation failures, demand surges, or unpredictable disturbances resulting in loss of one or more transmission lines or other system components. Generation failures and demand surges are handled by scheduling unused, but readily available reserved generation. Loss of one or more transmission lines is a more crucial issue. The distribution of power in the network, and hence the loading on any line, is determined by Kirchhoff's laws and depends on the total value and distribution of generation and load connected to the system; an individual generator has limited control over any power flows in the network. Although transmission networks are designed subject to stringent security and reliability requirements, unpredictable disturbances, such as lightning, may yet result in loss of facilities, thereby causing a redistribution of power throughout the network. If such redistributions result in surges and overloads of any remaining facilities, such as lines or transformers, it is possible that protective systems disconnect affected devices. If consecutive or cascading overloads persist in this manner a system black out is likely. Security and reliability address the prevention of such events and are in fact simply two sides of the same coin: From the PSO's perspective, the ability to avoid such cascading outages is called system security, while from the consumer's perspective, continuity of service is termed reliability.[12,17] The terms security and reliability are in general interchangeable and will henceforth be used as such.

System security is maintained by the PSO who verifies that the loading on a line or set of lines is below its loading limit. Loading limits are determined by the PSO off-line through contingency analysis. The PSO chooses a set of credible contingencies, and based on loading levels, determines the required transfer limits in order to maintain transient and dynamic stability in the event of unforeseen contingencies.[17]

1.2.4 Economic Dispatch

In the traditional vertically integrated environment, the PSO is responsible for achieving the most economic generation possible by performing what is called economic dispatch. Recall, in the traditional environment, transmission losses are simply considered to be part of the total generation required to meet the load and are addressed in the economic dispatch stage using penalty factors which reflect the impact of each generator on the transmission losses.[12,17]

Economic dispatch is performed every few minutes using real-time generation and load data. Small load deviations are addressed using participation factors obtained from the result of the economic dispatch to balance generation and load on a second by second basis.[12]

1.3 Power System Operation in a Deregulated Environment

1.3.1 Introduction

In the new deregulated regime, the issues of power balance and security have been further separated into what have been commonly referred to as “ancillary services”, which are defined as those basic electrical services required to support transactions between sellers and buyers of electricity and to maintain the integrity and reliability of the interconnected electrical networks.[1] Proponents of deregulation or decentralization claim that its purpose is to “benefit the industry and consumers to the tune of billions of dollars every year”. [1] The introduction of competition at all levels, including transmission, is considered to be an excellent method of promoting economic efficiency. Therefore, it is informative to understand the nature of the competition, the services being competed for, and how the power industry is being restructured to allow competition.

1.3.2 Ancillary Services and Transmission

In the United States, FERC Order 888 and the associated Pro Forma Access Tariff define ancillary services and state which must be offered by transmission providers and which must

be taken by transmission customers. The Pro Forma Tariff states concerning ancillary services that:[1]

Ancillary Services are needed with transmission service to maintain reliability within and among the Control Areas affected by the transmission service. The Transmission Provider is required to provide (or offer to arrange with the local Control Area operator as discussed below), and the Transmission Customer is required to purchase, the following ancillary services:

- 1. Scheduling, System Control, and Dispatch, and**
- 2. Reactive Supply and Voltage Control from Generation Sources**

The Transmission Provider is required to offer to provide (or offer to arrange with the local Control Area operator as discussed below) the following Ancillary Services only to the Transmission Customer serving load within the Transmission Provider's Control Area:

- 1. Regulation and Frequency Response**
- 2. Energy Imbalance**
- 3. Operating Reserve - Spinning, and**
- 4. Operating Reserve - Supplemental**

The Transmission Customer serving load within the Transmission Provider's Control Area is required to acquire these Ancillary Services, whether from the Transmission Provider, from a third party, or by self-supply. The Transmission Customer may not decline the Transmission Provider's offer of Ancillary Services unless it demonstrates that it has acquired the Ancillary Services from another source. The Transmission Customer must list in its application which Ancillary

Services it will purchase from the Transmission Provider.

If the Transmission Provider is a public utility providing transmission service but is not a Control Area operator, it may be unable to provide some or all of the Ancillary Services. In this case, the Transmission Provider can fulfill its obligation to provide Ancillary Services by acting as the Transmission Customer's agent to secure these Ancillary Services from the Control Area operator. The Transmission Customer may elect to:

- 1. Have the Transmission Provider act as its agent, or**
- 2. Secure the Ancillary Services directly from the Control Area operator, or**
- 3. Secure the Ancillary Services from a third party or by self-supply when technically feasible.**

In addition to the list given above, FERC also recognized real power loss replacement as a required ancillary service to compensate for transmission line losses.

To summarize, the services, in addition to their definitions as provided by FERC in Order 888, include:[1]

Scheduling, System Control, and Dispatch

Scheduling, System Control, and Dispatch represent the control area functions that assign generating units and transmission resources to supply anticipated loads, provide real-time control to maintain security, and bill for services. Dispatch includes the real-time balance of power by the Independent System Operator (ISO). Whereas the least cost dispatch is generally the goal in traditional system operation, this is not necessarily the case in a competitive environment, since the existence of transactions between suppliers and consumers dictate the generator output. Concerning Scheduling, System Control, and Dispatch, FERC states:

This service is required to schedule the movement of power through, out of, within, or into a Control Area. This service can be provided only by the operator of the Control Area in which the transmission facilities used for transmission service are located. Scheduling, System Control and Dispatch Service is to be provided directly by the Transmission Provider (if the Transmission Provider is the Control Area operator) or indirectly by the Transmission Provider making arrangements with the Control Area operator that performs this service for the Transmission Provider's Transmission System. The Transmission Customer must purchase this service from the Transmission Provider or the Control Area operator.

Reactive Supply and Voltage Control

Voltage control is necessary to maintain operating voltages within acceptable limits. Injection and absorption of reactive power is required for stability, and more specifically to protect against contingencies which could lead to voltage collapse. Reactive power supply and voltage control are grouped together since controlling one is paramount to controlling the other, since they are highly correlated. FERC says the following regarding this service:

In order to maintain transmission voltages on the Transmission Provider's transmission facilities within acceptable limits, generation facilities (in the Control Area where the Transmission Provider's transmission facilities are located) are operated to produce (or absorb) reactive power. Thus, Reactive Supply and Voltage Control from Generation Sources Service must be provided for each transaction on the Transmission Provider's transmission facilities. The amount of Reactive Supply and Voltage Control from Generation Sources Service that must be supplied with respect to the Transmission Customer's transaction will be determined based on the reactive power support necessary to maintain transmission voltages within limits that are generally accepted in the region and

consistently adhered to by the Transmission Provider.

There is also the related issue of local voltage regulation to address customer reactive power needs and alleviate power factor problems. FERC does not consider this a required service since each consumer generally has enough local information to correct these problems.

Regulation and Frequency Response

Regulation and Frequency Response involves the tracking of real time load fluctuations using automatic generation control for the purpose of maintaining real time power balance and operating frequency. FERC asserts:

Regulation and Frequency Response Service is necessary to provide for the continuous balancing of resources (generation and interchange) with load and for maintaining scheduled Interconnection frequency at sixty cycles per second (60 Hz). Regulation and Frequency Response Service is accomplished by committing on-line generation whose output is raised or lowered (predominantly through the use of automatic generating control equipment) as necessary to follow the moment-by-moment changes in load. The obligation to maintain this balance between resources and load lies with the Transmission Provider (or the Control Area operator that performs this function for the Transmission Provider). The Transmission Provider must offer this service when the transmission service is used to serve load within its Control Area. The Transmission Customer must either purchase this service from the Transmission Provider or make alternative comparable arrangements to satisfy its Regulation and Frequency Response Service obligation.

Energy Imbalance

Energy imbalance is the result of the inability of supplier and consumer to constantly and exactly match generation and load to pre-determined schedules. FERC defines the Energy

Imbalance service as follows:

Energy Imbalance Service is provided when a difference occurs between the scheduled and the actual delivery of energy to a load located within a Control Area over a single hour. The Transmission Provider must offer this service when the transmission service is used to serve load within its Control Area. The Transmission Customer must either purchase this service from the Transmission Provider or make alternative comparable arrangements to satisfy its Energy Imbalance Service obligation. To the extent the Control Area operator performs this service for the Transmission Provider, charges to the Transmission customer are to reflect only a pass-through of the costs charged to the Transmission Provider by that Control Area operator.

The Transmission Provider shall establish a deviation band of +/- 1.5 percent (with a minimum of 1 MW) of the scheduled transaction to be applied hourly to any energy imbalance that occurs as a result of the Transmission Customer's scheduled transactions(s). Parties should attempt to eliminate energy imbalances within the limits of the deviation band within thirty days or within such other reasonable period of time as is generally accepted in the region and consistently adhered to by the Transmission Provider. If an energy imbalance is not corrected within thirty days or a reasonable period of time that is generally accepted in the region and consistently adhered to by the Transmission Provider, the Transmission Customer will compensate the Transmission Provider for such service. Energy imbalances outside the deviation band will be subject to charges to be specified by the Transmission Provider.

Operating Reserves-Spinning Reserves

Spinning Reserves are required to correct for generation/load imbalances caused by generation and transmission outages. FERC states:

Spinning Reserve Service is needed to serve load immediately in the event of a system contingency. Spinning Reserve Service may be provided by generating units that are on-line and loaded at less than maximum output. The Transmission Provider must offer this service when the transmission service is used to serve load within its Control Area. The Transmission Customer must either purchase this service from the Transmission Provider or make alternative comparable arrangements to satisfy its spinning Reserve Service obligation.

Operating Reserves-Supplemental Reserves

Supplemental reserves are those which can be available within short time (30 min) to back up spinning reserves. Concerning the Supplemental Reserve Service, FERC states:

Supplemental Reserve Service is needed to serve load in the event of a system contingency; however, it is not available immediately to serve load but rather within a short period of time. Supplemental Reserve Service may be provided by generating units that are on-line but unloaded, by quick-start generation or by interruptible load. The Transmission Provider must offer this service when the transmission service is used to serve load within its Control Area. The Transmission Customer must either purchase this service from the Transmission Provider or make alternative comparable arrangements to satisfy its Supplemental Reserve Service obligation.

Real Power Loss Replacement

Real power loss replacement concerns the compensation of real transmission losses associated with power flows. Although its importance is recognized, FERC does not require that a transmission provider supply energy losses incurred in transmission, and therefore does not recommend that it be included in an open access transmission tariff. Loss replacement is a voluntary service which can be provided by any capable party.

The above services, required for system security and reliability, support the basic functions of producing and delivering electric power to customers. In the U.S., these services amount to approximately 12 billion dollars each year. [1] In addition to ancillary services, the equally important issue of transmission services must also be addressed. Traditionally, transmission charges have been bundled together with other electric utility costs as part of a single cost. The present trend towards decentralization requires the unbundling of all costs, and more importantly, the determination of a transmission fee. Clearly then, considering their role in system security, in addition to the yearly costs they represent, the issues of ancillary services and transmission are of utmost importance.

1.3.3 Operational Models in a Competitive Environment

The advent of competition in the power industry has created a need for restructuring of the system. There are two accepted and opposing sides to the debate concerning how this restructuring is to be accomplished, each side claiming itself as the route to economic efficiency: The Bilateral Model and the Poolco Model. In each model, any of the traditional PSO functions are performed by what is called the Independent System Operator (ISO). The ISO is so named to stress its independence from any market participant or class of participants, and its indifference regarding the performance of those participants.

The Bilateral Model is based on the principle that free market competition promotes economic efficiency. In an ideal bilateral model, market participants arrange trades, agreeing on the amount of generation, consumption, and financial terms with no involvement of interference by the ISO. Practically, the bilateral model is acceptable as long as the system remains secure. The ISO is allowed to intervene, either by curtailing or rejecting proposed transactions whenever security is endangered or violated.[6,7,12]

In the Poolco Model, all trades must pass through an intermediate entity known as the pool. The pool receives price bids from the suppliers and quantity bids from consumers, and the Independent System Operator (ISO) determines which bids are accepted as well as the pool

price. In the Poolco Model, the ISO is responsible for maintaining power balance, maintaining security, as well as coordinating transmission access and services, and therefore plays a role similar to that of the PSO in the traditional, vertically integrated environment. Clearly, the ISO's authority allows the achievement of efficient operation.[12]

In reality, the emerging market model in the United States is actually a combination of the Bilateral Model and the Poolco Model. While market clearing prices are established using a day-ahead generation wholesale auction, bilateral contracts are also envisioned as a means of hedging future generation prices.[1]

1.4 Historical Review of Loss and Line Loading Allocation

1.4.1 Introduction

There have been several proposed methods for loss and line flow allocation. As a consequence of the highly nonlinear nature of the relationship between the power injections and loads, and more specifically between individual bilateral transactions, the proposed allocation procedures have been based on approximate or heuristic methods. A list of the more popular methods includes:

1. Coordinated Multilateral Trade Model
2. Localized Response Based Loss Allocation
3. Quadratic Loss Approximation
4. Contract Path with Postage Stamp Rate
5. MW-Mile Technique

Items 1, 2, and 3 address the loss approximation only, while 4 and 5 deal with the issue of line-loading allocation. Each item is discussed in greater detail below.

1.4.2 Coordinated Multilateral Trade Model

The purpose of the Coordinated Multilateral Trade Model (CMTM) is to provide a new operating paradigm in which the decision mechanisms regarding economics and system security and reliability are separated. More specifically, the CMTM is claimed to provide generators and consumers with the ability to make economic decisions, that is, construct private, profitable multilateral trades, with minimal intervention by the power system operator (PSO), a profitable trade being one for which the cost of generation is less than the consumer benefit. Furthermore, Wu and Varaiya state that the proposed CMTM also solves the loss allocation problem, which previously weakened the bilateral model.[11,12]

Whereas both the information structure and decision-making authority required to achieve power balance and security are centralized in the form of a PSO in the traditional operating paradigm, such centralization is inconsistent with the goals of a competitive environment, since such a market demands that sellers and consumers be allowed to perform any desired profitable trades. A PSO, with its traditional decision making authority, has no incentive for finding the most profitable operating point, and in fact, is only concerned with security. Consequently, it is suggested that more authority be given to individual market participants through proper coordination and information structure, so that security issues can be addressed at the market level, and incorporated while economic decisions are being made. The proposed information structure consists of two main components: line flow sensitivity vectors and loss vectors.[12]

The i^{th} element of a line flow sensitivity or loading vector represents the increase in transmission loading on a congested line occurring if one megawatt of power is injected into the network at bus i . The loading vector allows participants to verify if a planned trade is a *feasible trade*, that is, whether or not a proposed trade will violate flow on any congested lines. Note that a corresponding loading vector is broadcast for each congested line. If a proposed trade results in additional congestions, the loading vectors corresponding to these

lines are broadcast for future trades.

Regarding losses, it is suggested that a quadratic approximation procedure, which is usually correct within a few percent, be used to calculate transmission losses. If the system operating point is currently defined by the set of net power injections, (P_0, P) , where P_0 is the slack bus injection and P is a vector of injections corresponding to buses 1 through N , then it is shown [12] that the effect of an additional trade, $(\Delta P_0, \Delta P)$, on transmission losses can be written as,

$$\Delta P_L = \langle R(\theta), \Delta P \rangle + \frac{1}{2} \Delta P^T Q(\theta) \Delta P \quad (1.1)$$

where,

$$\begin{aligned} R(\theta) &= \bar{1} + \left(\frac{\partial F}{\partial \theta}(\theta) \right)^{-T} \left(\frac{\partial f_0}{\partial \theta} \right) \\ Q(\theta) &= \left(\frac{\partial F}{\partial \theta}(\theta) \right)^{-T} H(\theta) \left(\frac{\partial F}{\partial \theta}(\theta) \right)^{-1} \\ H(\theta) &= H_0(\theta) + \dots + H_N(\theta) \end{aligned} \quad (1.2)$$

and $H_k(\theta)$ is the hessian matrix of the k -th power flow equation, $F_k(\theta) = P_k$, and θ is the vector of voltage phase angles.

Since it is more desirable to obtain a loss formula which does not depend on the operating point, θ , Wu and Varaiya show that losses can be further approximated using,

$$P_{Lk} = \langle R(0), q_k \rangle + \left(\sum q_m \right)^T Q(0) q_k \quad (1.3)$$

In addition to being independent of voltage phase angles, θ , the matrix, $Q(0)$, provides an

understanding of the interaction of trades in terms of their effect on system losses.[12]

Briefly, the application of the CMTM proceeds as follows [12]:

Step1 Initialization

Brokers arrange loss included trades q_k . Let $q^0 = \sum q_k$.

Step 2 Curtailment

If q^0 is not feasible, the power system operator (PSO) curtails the trades (uniformly) to a point where the resulting injections q are feasible.

Step 3 Announcement

If lines k_1, \dots, k_m are congested at q , the PSO announces the loading vectors, n_k ,
 $k = k_1, \dots, k_m$.

Step 4 Trading

If a profitable trade in the feasible directions is found, a broker arranges it. The broker uses n_k to determine whether a trade is in the feasible direction. If no profitable trade is found, go to step 6.

Step 5 Feasibility

If the trade is infeasible, let the PSO curtail the trade and go to step 3. If the trade is feasible, let the PSO fulfill it and go to step 4.

Step 6 Termination

Stop.

While it is true that the CMTM's use of sensitivity vectors allows security and reliability issues to be addressed at the market level, they also unfortunately restrict transactions

between suppliers and consumers. Since it is possible for two transactions to have opposing effects on line flows, it is possible that the sensitivity vectors result in the rejection of what could have been a feasible trade. Therefore, the feasibility of a proposed trade should be judged based on the feasibility of the aggregate sum of all proposed transactions.

1.4.3 Heuristic Loss Approximation

In this method, a loadflow model is used to calculate the incremental losses, L_j , incurred by each transaction if it were the last one added to the system. The sum of these incremental losses, P_L^* , is scaled up or down to match the actual total losses, P_{loss} , obtained with all transactions present. Mathematically, the algorithm can be represented as,

$$q = \frac{P_{loss}}{P_L^*} = \frac{P_{loss}}{\sum_j L_j} \quad (1.4)$$

$$P_{L_j} = qL_j$$

where P_{L_j} is the loss attributed to transaction j for charging purposes.[1] It should be noted that the above scheme may result in negative losses; this topic is not addressed and the manner in which this issue is resolved remains unclear.

1.4.4 Transmission Loss and Localized Response

It is known that the change in voltage phase angles and phase angle differences across transmission lines decrease monotonically as the electrical distance from a (single) triggering event increases, a property defined as localized response. If changes in power injections resulting from the introduction of multiple simultaneous bilateral transactions are electrically distant from each other, then the localized response property allows individual transmission losses to be approximated in a localized manner, independent of other transactions. The algorithm for computing real power transmission losses is based on linear, decoupled real power load flow equations, and only requires that the change in power injection at node i ,

ΔP_i , and the network parameters of the system be available.[9]

The proposed loss computation procedure can be characterized by its tier-based bus numbering scheme, its use of second order loss formulae, as well as its localized response properties.

A bus where a change in power injection occurs is referred to as tier one for the purpose of loss calculation. Any buses directly connected to tier one belong to tier two, while any remaining buses directly connected to tier two are included in tier three. The numbering process continues in this fashion until all buses belong to a tier. In essence, each bus has its own unique tier based numbering scheme, tier one being itself. This re-enumeration allows for a convenient block tri-diagonal matrix representation of the system relative to the location of a single triggering event at a bus, that is, tier one,

$$\begin{bmatrix} B_{11} & B_{12} & 0 & \dots & \dots & \dots & 0 \\ B_{21} & B_{22} & B_{23} & \dots & \dots & \dots & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & \dots & \dots & B_{(n-1)(n-2)} & B_{(n-1)(n-1)} & B_{(n-1)n} \\ 0 & 0 & \dots & \dots & 0 & B_{n(n-1)} & B_{nn} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_{n-1} \\ \delta_n \end{bmatrix} = \begin{bmatrix} \Delta P_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.5)$$

The tri-diagonal structure arises from the fact a general tier i can only be connected to tier $(i-1)$ and $(i+1)$. It is important to note that since the DC loadflow formulation is linear, the effects of all changes on the system phase angles can be computed by superposition. In fact, linearity also allows the effects of the generation and load component of a bilateral transaction to be computed independently. The combined effect of all changes can be approximated by summing the effects of each individual change. This issue will be addressed shortly.

The loss calculation procedure uses a second-order function to approximate the losses on a

line connecting buses i and j ,

$$P_{L_{ij}} = G_{ij}(\delta_i - \delta_j)^2 = G_{ij}\delta_{ij}^2 \quad (1.6)$$

where the definition $\delta_{ij} = \delta_i - \delta_j$ has been introduced. Defining $\Delta\delta_{ij}$ as the change in phase angle difference resulting from the addition of multiple, simultaneous bilateral transactions, the updated transmission losses can be written as,

$$\begin{aligned} P_{L_{ij}} &= G_{ij}(\delta_{ij} + \Delta\delta_{ij})^2 \\ &= G_{ij}(\delta_{ij}^2 + 2\delta_{ij}\Delta\delta_{ij} + \Delta\delta_{ij}^2) \end{aligned} \quad (1.7)$$

It is stated in [9] that in a competitive industry, the complete set of phase angle differences, $\{\Delta\delta_{ij}\}$, will not be available, making it important to develop a method of determining losses locally. Recall that $\Delta\delta_{ij}$ is estimated using a linearized P- δ model. Consequently, superposition holds and the total change in phase angle difference across any transmission line, $\Delta\delta_{ij}$, can be calculated as the sum of the changes resulting from each individual transaction,

$$\Delta\delta_{ij} = \Delta\delta_{ij,1} + \dots + \Delta\delta_{ij,k} \quad (1.8)$$

where $\Delta\delta_{ij,k}$ is the change in angle difference across the line connecting buses i and j resulting from a change in injection at bus k . Hence, expanding the terms in (1.7) yields,

$$\begin{aligned} 2\delta_{ij}\Delta\delta_{ij} &= 2\delta_{ij}(\Delta\delta_{ij,1} + \dots + \Delta\delta_{ij,k}) \\ \Delta\delta_{ij}^2 &= \Delta\delta_{ij,1}^2 + \dots + \Delta\delta_{ij,k}^2 + \\ &\quad 2\Delta\delta_{ij,1}\Delta\delta_{ij,2} + \dots + 2\Delta\delta_{ij,1}\Delta\delta_{ij,k} + \dots \end{aligned} \quad (1.9)$$

Assuming that the changes in power injections are electrically distant, $\Delta\delta_{ij}^2$ is approximated as,

$$\Delta\delta_{ij}^2 = \Delta\delta_{ij,1}^2 + \dots + \Delta\delta_{ij,k}^2 \quad (1.10)$$

More specifically, for a particular line connecting buses i and j, if the phase angle change arising as a result of an injection at bus m, $\Delta\delta_{i,m}$ is significant, then assuming all other injection are sufficiently far away, the phase angle differences, $\Delta\delta_{i,k}$ for $k \neq m$ are negligible. Consequently, the cross terms, $\Delta\delta_{i,j}\Delta\delta_{i,k}$, can be omitted for $j, k \neq m$, since at least one of the terms involved in the product is close to zero.[9]

Hence, the transmission line loss across line i-j can be written,

$$\begin{aligned} P_{L_{ij}} &= G_{ij}[\delta_{ij}^2 + 2\delta_{ij}\Delta\delta_{ij,1} + \dots + 2\delta_{ij}\Delta\delta_{ij,k} + \dots + \Delta\delta_{ij,1}^2 + \dots + \Delta\delta_{ij,k}^2] \\ &= G_{ij}\delta_{ij}^2 + G_{ij}(2\delta_{ij}\Delta\delta_{ij,1} + \Delta\delta_{ij,1}^2) + \dots + G_{ij}(2\delta_{ij}\Delta\delta_{ij,k} + \Delta\delta_{ij,k}^2) \\ &= P_{L_{ij}}(\delta_{ij}) + \Delta P_{L_{ij}}(\Delta\delta_{ij,1}) + \dots + \Delta P_{L_{ij}}(\Delta\delta_{ij,k}) \end{aligned} \quad (1.11)$$

where $P_{L_{ij}}(\delta_{ij})$ are the base case losses to be provided by the OASIS and $\Delta P_{L_{ij}}(\Delta\delta_{i,k})$ is the change in transmission losses resulting from a change in power injection at bus k.

The application of the algorithm for estimating real power losses proposed in [9] can be summarized in four steps.

The first step is to re-enumerate the network buses using the tier based scheme previously described to obtain the block tri-diagonal B matrix.

Once all the data is available, the second step is for a player to compute changes in bus voltage phase angles based on the variation in bus injection at his/her bus, independent of other system wide changes. This is accomplished using a forward-backward substitution

algorithm to solve (1.5) for the phase angle change at tier one, $\Delta\delta_1$, which can be shown to be,

$$\begin{aligned}\Delta\delta_1 &= (B_{11} - B_{12}\bar{B}_{22}^{-1}B_{21})\Delta P_1 \\ \Delta\delta_j &= (-\bar{B}_{jj}^{-1}B_{j,j-1})\Delta\delta_{j-1}\end{aligned}\quad (1.12)$$

for $j = 2, \dots, N$, where,

$$\bar{B}_{kk} = B_{kk} - B_{k(k+1)}B_{(k+1)(k+1)}^{-1}B_{(k+1)k}\quad (1.13)$$

for $k = 2, \dots, (N-1)$ and,

$$\bar{B}_{NN} = B_{NN}\quad (1.14)$$

After the change in node angles are known, the variations in phase angle differences, $\Delta\delta_{ij}$, can be computed.

It is important to note that the derivations presented above facilitate the computation of voltage angle changes for the purpose of determining the effect of a change in real power injection at one bus on real power losses. Bilateral transactions are composed of both a generation and a load component and, therefore, result in a variation in injection at two buses. The last two steps address a practical application of the algorithm for approximating losses caused by such transactions.

The third step is to compute the change in real power losses caused by the addition of a load. For example, suppose a load at bus i demands ΔP_k MW. Following the calculation of the change in phase differences across all transmission lines resulting from the introduction of this

load using the tier based re-enumeration and the forward-backward substitution procedure described earlier, the line by line variation in real power losses can be computed using (1.15),

$$\Delta P_L(\Delta\delta_{j,1}) = G_{ij}(2\delta_j\Delta\delta_{j,1} + \Delta\delta_{j,1}^2) \quad (1.15)$$

where as before, $\Delta\delta_{ij}$ is the change in phase angle difference across the line connecting buses i and j caused by a change in power injection at node 1. Denoting the sum of these incremental, line by line losses as ΔP_{Lk} , the load located at bus k must contract to buy $(\Delta P_k + \Delta P_{Lk})$ MW.

The last step is for the corresponding generator at bus m to compute the losses, ΔP_m , it introduces into the system by generating $(\Delta P_k + \Delta P_{Lk})$ MW.

In conclusion, it is important to note that the practical application of the proposed loss approximation algorithm is questionable. First of all, accurate results are dependent on changes in injections being electrically distant; an unlikely scenario in a practical system, implying that the generation and load buses involved in a bilateral transaction must also be electrically distant. It is admitted in [9] that results are poor if multiple changes occur in close proximity to each other, since interaction effects are not negligible in such cases. Hence, the experimental models and examples developed to illustrate the validity of the localized response property for the purpose of estimating real power losses are unrealistic, casting doubt on its usefulness in allocating transmission losses accurately and fairly in a competitive environment. Furthermore, the development of the loss approximation formula and, in particular, the omission of the cross terms in equation (1.10) is somewhat arbitrary. The cross terms and the quadratic terms are actually of the same order of magnitude. In fact, some quadratic terms are actually smaller than the cross terms. Hence, even under the assumption that the localized response property is valid, the theoretical development is not entirely justified.

1.4.5 Contract Path With Postage Stamp Rate

Using contract paths in conjunction with a fixed postage stamp rate has been recently proposed as a method of allocating line-loading and line usage to individual bilateral transactions.

The method of contract paths applies to transactions utilizing multiple transmission systems for power transfer. The notion of a contract path is based on the assumption that power flows through a specified path from source to sink. Although it has been proven that contract paths do not reflect the actual flow in the system, its relative simplicity has made it the prevailing method of flow allocation. A postage stamp transmission rate is a fixed rate generally based on the peak system MW capacity and total fixed transmission costs associated with the corresponding transmission system. A utilization fee is calculated by multiplying the postage stamp rate by the peak MW transfer involved in a transaction.

The contract path/postage stamp flow allocation rate is calculated in two steps. First, the least cost electrical path between the source and sink points corresponding to a transaction is defined as the contract path for that power transfer. The transaction is then charged a postage stamp rate, one for each of the transmission systems traversed along the defined contract path.

The contract path/postage stamp flow allocation procedure is deficient in two respects. Firstly, the method does not differentiate between users of costly or inexpensive transmission facilities. Secondly, this allocation scheme suffers from the loop flow problem, often causing the curtailment of legitimate transactions for the purpose of accommodating flows of unknown origin. Moreover, transmission providers remain uncompensated for such loop flows.

1.4.6 MW-Mile Method

The MW-Mile method attributes a transmission usage rate using the concept of transmission

line capacity use, defined as a function of transaction magnitude and the distance travelled.

First, for each individual transaction, a DC load flow algorithm is used to approximate all the real power flows on all transmission lines in the system. Defining P to be the vector of given real power injections, excluding the slack bus, and δ to be the vector of voltage angles, excluding the reference bus (usually taken to be the same as the slack), and B to be the imaginary component of the network admittance matrix, that is, the susceptance matrix, with column and row corresponding to the slack and reference buses deleted, then the voltage phases at each bus, represented by δ , can be obtained by solving,

$$P = B\delta \quad (1.16)$$

Assuming all voltage magnitudes are 1 per unit, the flow magnitude on any transmission line connecting buses i and j can then be approximated by,

$$F_{ij} = b_{ij} |\delta_i - \delta_j| \quad (1.17)$$

where b_{ij} is the susceptance of the corresponding line.

The cost associated with a transaction is determined as,

$$C = \sum L_{ij} W_{ij} F_{ij} \quad (1.18)$$

where L_{ij} is the length and W_{ij} a weighting factor representing the cost per unit capacity of the line connecting buses i and j . As shown, the cost is calculated by summing over all transmission lines, using the results obtained using the DC load flow solution.

Since the sum of the individual transaction costs obtained heuristically by repeated application

of the DC loadflow algorithm may not equal the actual fixed cost of the system, a scaling factor p is calculated, where

$$p = \frac{(\text{Total Fixed Cost})}{\sum_k C_k} \quad (1.19)$$

and C_k is the cost corresponding to the k^{th} transaction.

The advantage of this scheme is that it guarantees the recovery of all fixed costs, the scaling factor p forcing the sum of the payments to equal the total fixed costs. The primary disadvantage of this scheme stems from the fact that while the DC loadflow formulation is reasonably accurate for (nearly) flat voltage profiles, it is insufficient in the general case. Hence, transmission users will be charged unrealistic rates which fail to reflect the exact impact on the network of the corresponding transactions.

1.4.7 MIT Flow Allocation Method

The first step in this method is to determine the complex line flows resulting from the presence of all network transactions using an AC loadflow algorithm. The complex n -vectors corresponding to complex power injection, complex voltage, and complex current are denoted S , V , and I , respectively. Repeated solving the load flow for each individual transaction provides the set of corresponding voltage and current vectors, $\{V_i\}$ and $\{I_i\}$, where V_k and I_k are the voltage and current vectors attributed to transaction k . The line flow on line i - j corresponding to transaction k can subsequently be computed as,

$$F_{ij}^k = V_i^{*k} I_{ij}^k \quad (1.20)$$

The allocation of fixed cost to transaction k is then computed as,

$$FC^k = \sum CC_{ij} \frac{|F_{ij}^k|}{F_{ij}^{tot}} + \sum CC_{ij} \frac{|F_{ij}^{int}|}{F_{ij}^{tot}} \frac{peak^k}{\sum_k peak^k} \quad (1.21)$$

where CC_{ij} is the carrying charge for the line connecting buses i and j , $|F_{ij}^k|$ is the absolute value of complex power flow through the line resulting from transaction k , and F_{ij}^{tot} is the absolute value of the total complex flow on the line resulting from all transactions,

$$F_{ij}^{tot} = \sum_k |F_{ij}^k| + |F_{ij}^{int}| \quad (1.22)$$

F_{ij}^{int} is the flow component resulting from interactions between transactions and is obtained by multiplying the voltage and current corresponding to different transactions, while $peak^k$ is the peak MW transfer involved in transaction k .

The algorithm described above uses non-physical methods of evaluating the effects of, and interactions between, individual transactions. First, since the power flow equations are highly nonlinear, the effects of simultaneous transactions cannot, in general, be approximated by superposition. Consequently, the voltages and currents, V_k and I_k , and the resulting power flows are inaccurate. It should be noted that the flow values utilized in the cost allocation process are non-physical; that is, they do not exist in reality. The actual net line flows resulting from a set of simultaneous bilateral transactions are (approximately) those obtained using a full AC load flow model. Using the sum of the absolute values of the line flows obtained by superposition is debatable since this value need not reflect the actual net flow. Consideration of a simple two bus system connected by one transmission line reveals that while the line flow attributed to two individual transactions, one at each bus, using the above algorithm can be substantial, the actual net flow may be close to zero. Furthermore, the

suggestion that the flow component resulting from the interaction between transactions at buses i and j , respectively, can be calculated by cross multiplying the voltage and current vectors obtained by repeatedly applying an AC loadflow model to each individual transaction is questionable; bus voltages and line currents are each a function of all transactions. In essence, the algorithm uses m different operating points to determine the cost allocation for m corresponding bilateral transactions, whereas, on the contrary, the actual operating point is completely determined by the combined simultaneous effect of all transactions.

1.4.8 United Illuminating Company Allocation and Charging Procedure

United Illuminating's (UI) proposal allocates to each generator a transmission charge representing its relative share of the regional system costs. The relative share associated with each generator is calculated by converting its MW-mile product to a percentage of the sum of all generators' MW-miles of line use.[4]

UI's allocation procedure is based on the well established result that each generator's use of the transmission system is primarily dependent on the impedance of the transmission grid and is virtually independent of the load level or location of the load. Specifically, for a particular transmission grid configuration, the percentage of a generator's output that flows on any line will remain the same regardless of generation output level, load level, load distribution, or dispatch of other generators.

The allocation is based on a two-step load flow testing procedure: An initial loadflow corresponding to an individual generator operating at full output defines the reference case, while a second analysis for the case of the generator operating at zero output provides the additional measurements required to calculate the generator's corresponding distribution factors, that is, what percentage of the generator's output flows on each transmission line. It should be noted that all system loads are scaled uniformly by an amount equal to the change in the (test) generator's output to maintain the balance of load and generation without changing any variables other than the one being measured. That is, loads are adjusted so that

all other generator outputs are unchanged, or equivalently so that all line flows associated with the other generators remain unchanged.

The differences in line flows realized using the two-step load flow tests are used to create a static set of distribution factors that reflect the impact of the test unit. The product of the distribution factor for a particular line and the full output of the generator is then multiplied by the line length producing a MW-mile value for that line, that is,

$$MW-Mile_{line} = (Distribution\ Factor) \times (Full\ Gen\ Output) \times (Line\ Length) \quad (1.23)$$

The sum of the MW-miles for all lines represents the total MW-mile impact of the generator.

$$MW-Mile_{gen\ i} = \sum_{all\ lines} (MW-Mile_{line\ i}) \quad (1.24)$$

The above analysis is repeated for all generators in the system and the resulting MW-mile values summed to determine the total use of the transmission system. Each generator is allocated a corresponding percentage share of total system costs according to the equations,

$$\begin{aligned} \% \text{ Share}_{gen\ i} &= \frac{MW-Mile_{gen\ i}}{\sum_{all\ gen} MW-Mile_{gen\ i}} \\ Cost_{gen\ i} &= (\% \text{ Share}_{gen\ i}) \times (Total\ Transmission\ Cost) \end{aligned} \quad (1.25)$$

Although UI's method for determining the relative costs associated with each generator may be reasonable, it does not address the issue of bilateral contracts. The ultimate question which must be addressed in a competitive environment is how to distribute costs over all transactions. UI's proposed method at least requires an additional mechanism for distributing each generator's cost over its corresponding contracts. It is the author of the present thesis'

belief that this could possibly be accomplished by simply distributing a generator's portion of the line flows proportionally over all the generator's transactions. That is, if a generator produces a 100 MW line flow and a particular transaction contributes fifty percent of the generator's output, then the transaction should be allocated fifty percent of that line flow or 50 MW.

It should also be remarked that the assumption that the line flow distribution factors remain constant is essentially equivalent to utilizing a DC loadflow formulation to allocate line loading to individual generators or transactions. Unfortunately, the DC loadflow approximation is only valid when the voltage profile is flat, line resistances are negligible relative to line reactances, and bus phase angle differences are close to zero. As will be shown later, the DC loadflow solution can become very inaccurate, even when the deviations are not very substantial. The approximation may, therefore, be inappropriate considering that transmission services can amount to billions of dollars each year.[1]

1.5 Statement of Thesis

A sound mathematical and physical understanding of the impact of individual bilateral transactions on system losses and line flows is required in a competitive, decentralized environment. Otherwise, market participants run the risk of being charged unfairly for the use of the power system. The search for simplicity has promoted the use of heuristic and approximate methods of loss and line flow allocation and, in most cases, particularly concerning line loading allocation, the proposed methods, in particular the use of *contract paths*, favour power and transmission providers. That is, no attempt is made to determine the exact impact of each transaction, the main concern being that suppliers and transmission providers recover their costs. Consumers are inadvertently cheated by such recommendations.

In a traditional, vertically integrated power system environment, all facilities were generally

owned by one party, that is the utility. Consequently, it was to the utility's advantage to operate at minimum cost, and unnecessary to determine each individual user's share of the total cost; every user was simply charged a fixed rate, regardless of the relative location of generators and loads. In particular, it was unnecessary to allocate transmission losses and transmission line use to individual power transactions, since all facilities were the property of one entity.

Under open access, market-driven transactions have become the new independent decision variables defining the behavior of power networks. Generally, a transaction is characterized by a power transfer from a source to a sink, across a network of transmission lines. The source may be a generating area made up of a number of generators injecting power into a set of buses and, similarly, the sink may be constituted by a group of load buses, each receiving a fraction of the total transfer. However, in its most basic form, a transaction is a transfer of power between two buses, regardless of the competitive model being used. This is so since transfers between sources and sinks, no matter how complex, can always be decomposed into the sum of several bus-to-bus transactions. [18]

The power transferred as a result of a transaction does *not* follow a predefined path, but rather spreads throughout the network following the laws of nature - Kirchhoff and Ohm - according to the type of network, mode of operation and kind of transaction. The so-called "loop flows" experienced by power networks are simply a manifestation of this phenomenon - one which frequently forces transmission providers to curtail legitimate transactions in order to accommodate flows of unknown origin. Understanding the impact of transactions on system losses and line-loading is, therefore, crucial for the secure operation of the network as well as to be able to establish equitable tariffs based on actual network use. Clearly, power suppliers and loads do not wish to subsidize a competing party's system losses or transmission use, especially considering that these services amount to billions of dollars per year.

The main difficulty in allocating a fraction of the system losses or line loading to a particular

transaction is the highly nonlinear nature of their relationship. One can circumvent this difficulty by resorting to linear approximations; however, the corresponding results are inaccurate and, therefore, inequitable. The theory presented in this thesis is based on the argument that the loss allocation for an infinitesimal transaction increment can always be computed exactly. This then leads to a set of governing differential equations whose solution yields the exact nonlinear loss allocation.

The goal of this thesis is to present a general theory of loss and line-loading allocation for individual bus-to-bus transactions. The results are valid under the nonlinear load flow equations and do not require any approximations.

Chapter 2

Theory of Loss and Line Flow Allocation

2.1 Introductory Remarks

Under open access, market-driven transactions have become the new independent decision variables defining the behavior of power networks. Generally, a transaction is characterized by a power transfer from a source to a sink across a network of transmission lines. The source may be a generating area made up of a number of generators injecting power into a set of buses and, similarly, the sink may be constituted by a group of bus-loads each receiving a fraction of the total transfer. However, in its most basic form a transaction is a transfer of power between two buses. This is so since transfers between sources and sinks, no matter how complex, can always be decomposed into the sum of several bus-to-bus transactions. [18]

The power transferred as a result of a transaction does *not* follow a predefined path, but rather spreads throughout the network following the laws of nature - Kirchhoff and Ohm - according to the type of network, mode of operation and kind of transaction. The so-called "loop flows" experienced by power networks are simply a manifestation of this phenomenon - one which frequently forces transmission providers to curtail legitimate transactions in order to accommodate flows of unknown origin. Understanding the impact of transactions on system losses and line-loading is, therefore, crucial for the secure operation of the network as well as to be able to establish equitable tariffs based on actual network use.

The goal of this chapter is to present a general theory of loss and line-loading allocation for individual bus-to-bus transactions. The results are valid under the nonlinear load flow equations and do not require any approximations.

The main difficulty in allocating a fraction of the system losses or line loading to a particular transaction is the highly nonlinear nature of their relationship. One can circumvent this

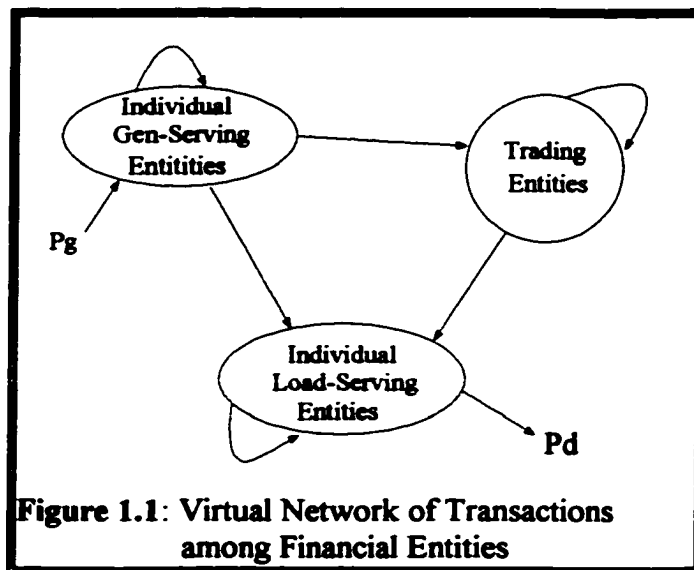
inaccurate and, therefore, inequitable. This chapter begins by introducing the mathematical framework required for analyzing the effect of bilateral transactions on a power system. In this context, the notions of contractual losses and flows and their impact on the network are introduced. The theory presented here is based on the argument that the loss allocation for an infinitesimal transaction increment can always be computed exactly. This then leads to a set of governing differential equations whose solution yields the exact nonlinear loss allocation. The relationship between the proposed allocation theory and incremental transmission loss (ITL) coefficients is also discussed. An in-depth development of the mathematical theory follows the introduction of the relevant terminology.

2.2 Framework for the Analysis of Power Transactions

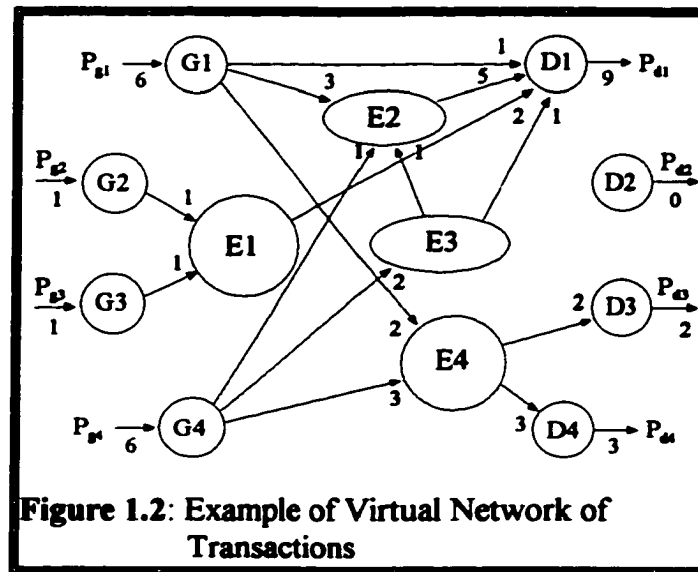
Under open access, bilateral, market-driven transactions between entities, that is, buyers and sellers of electricity, have become the new independent decision variables defining the behaviour of power networks. In the traditional, vertically-integrated environment, the notion of individual transfers of power between generators and loads was nonexistent, since the entity supplying power, that is, the utility, owned all generation and transmission facilities. More specifically, all transactions were essentially with the utility which charged consumers a flat rate and operated at minimum cost, while maintaining power balance and security. In contrast, the advent of competition in the power industry introduces the possibility of several types of transactions. For example, individual generators can sell power directly to loads, the pool, or to trading entities, which may subsequently also trade with loads or among themselves. Transactions between market participants, although essentially financial agreements, both directly and indirectly determine physical network operating parameters such as loads, reserves, generation levels, power flows, voltage levels, losses, and operational costs. Consequently, a mathematical framework for modeling and analyzing transactions is required to operate both efficiently and reliably, as well as to determine the exact impact of individual transactions on the network for the purpose of allocating costs.[18]

As previously mentioned, transactions are essentially financial agreements involving market participants. As such, a set of financial transactions involving the buying and selling of electrical energy can be viewed as defining a virtual network of transactions, which rather than being composed of real physical devices conducting power flows from generators to loads, simply models power transactions between financial entities. The most general transactions network, depicted in figure (1.1), consists of three types of financial entities:

- (1) Individual generator serving entities representing the selling interests of individual physical generators.
- (2) Individual load serving entities representing the buying interests of retail loads.
- (3) Trading entities which can be further decomposed into three types:
 - (a) group generator-serving entities, serving the selling interests of groups of individual generators.
 - (b) group load-serving entities, serving the buying interests of groups of individual loads.
 - (c) pure trading entities, such as marketers, trading for their own profit with individual or group entities of any kind.



For example, under open access, a utility owning generators can act as a group generator-serving entity, while an independent power producer (IPP) represents an individual generator-serving entity. Likewise, loads can also join together, allowing the possibility of both individual and group load-serving entities, a power pool being an example of the latter. Lastly, marketers are examples of pure trading entities which can buy or sell to any other entity. Figure (1.2) is an example of a virtual network of transactions. G1 through G4 are individual generator-serving entities, D1 to D4 are individual load-serving entities, E1 is a group generator-serving entity, while E4 is a group load serving entity. E2 and E3 represent pure trading entities. Note that the total power injected at any node is equal to the that leaving the same node.

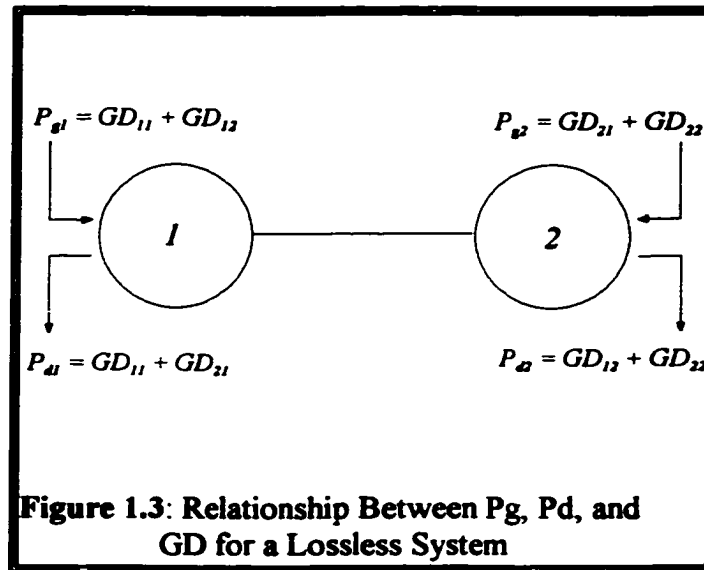


The above framework must be represented in more rigorous mathematical terms for analytical purposes. To this end, the power transactions between entities are represented by a set of matrices: GD , GE , ED , and EE . These matrices are briefly defined below.

- (1) GD_{ij} = The real power generated by a generator at bus i for a load at bus j .
- (2) GE_{ij} = Power sold by a generator at bus i to trading entity j .
- (3) ED_{ij} = Power sold by trading entity i to a load at bus j .
- (4) EE_{ij} = Power sold by trading entity i to trading entity j .

It is shown in [18] that the set of transactions between entities can be represented by an equivalent transactions matrix, GD_{eq} , since the fundamental building block of all power transactions, no matter how complex, is a transfer of power between a generator and a load. Consequently, for simplicity, and without loss of generality, it will be assumed throughout this thesis that the virtual network of transactions is completely described by the transaction

matrix, GD.



As previously stated, financial transactions both directly and indirectly determine physical network operating parameters. For example, assuming that real power losses are zero, the equivalent transactions matrix, GD, defines both the network generation and load quantities, as shown in figure (1.3). Figure (1.3) depicts the four possible transactions which may exist in a two-bus system having both a generator and a load at each bus. First of all, generators may sell power to local loads; these transactions are represented by GD_{11} and GD_{22} . Secondly, each generator can supply power for a load at the other bus; these transactions are represented by GD_{12} and GD_{21} . Note that each quantity appears twice to represent both the generation and the load components of a transaction.

Unfortunately, physical power systems are not lossless. Consequently, the virtual network of transactions and the equivalent transactions matrix, GD, are insufficient for the purpose of completely describing the effects of each transaction. Hence, an analytical method for determining the exact loss component corresponding to each transaction is also required.

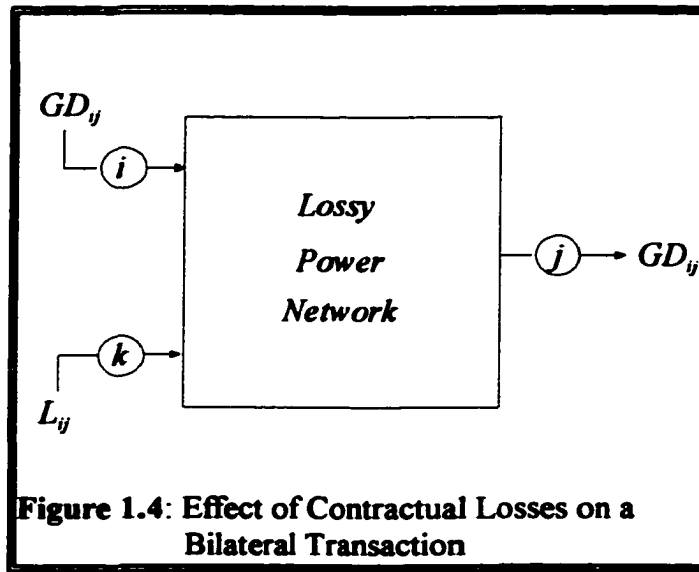
2.3 Transactions and Contractual Loss Components

As will be shown later, all transactions, with the exception of those involving a generator and load at the same bus (i.e., GD_{ii}), have an associated non-zero contractual loss component. Consequently, a loss matrix, L , must be introduced for the purpose of tracking contractual losses. Since losses, in essence, also represent transfers of power between two buses, they too can be interpreted as bilateral transactions. Furthermore, in the event that loss replacement is also an ancillary service and open to competition, the contractual losses corresponding to the transaction GD_{ij} can in theory be supplied by an arbitrary bus; that is, contractual losses need not be supplied by the same generator supplying the corresponding transaction. As a result, in addition to tracking the value of each loss component, it becomes necessary to store the index of the bus responsible for supplying the losses, L_{ij} , corresponding to the transaction, GD_{ij} . Hence, the loss allocation process requires the existence of both a loss matrix, L , and a loss index matrix, U , defined as follows:

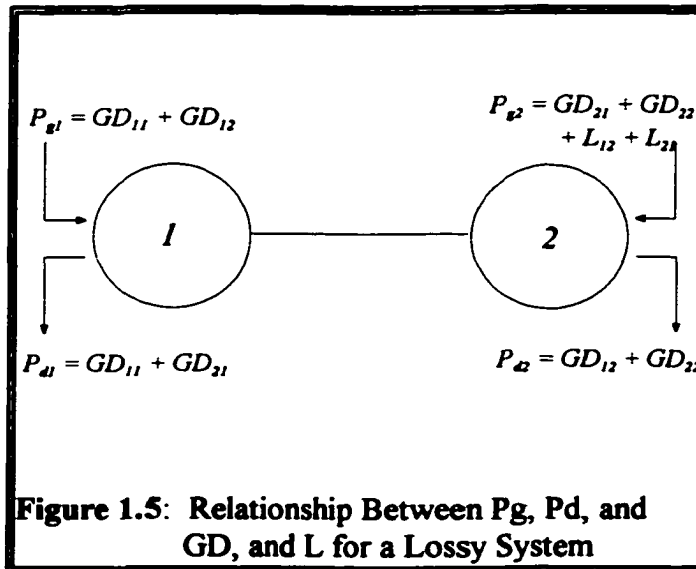
L_{ij} = the value of the contractual losses assigned to the transaction GD_{ij}

U_{ij} = the index of the bus supplying L_{ij}

The concept of loss allocation can be better understood by referring to figure (1.4). As stated before, each transaction, GD_{ij} , describes both a generation and load component, and is, therefore, injected at one bus and dissipated at another. Since a real power system is lossy, these quantities are not sufficient to guarantee power balance. Therefore, L_{ij} must be injected at bus $U_{ij} = k$. It is important to note that k can equal i or j . That is, it is possible that the corresponding contractual losses are supplied by the generator or load bus involved in the transaction GD_{ij} .



The interpretation of the loss allocation process becomes clearer upon the application of figure (1.4) to a physical two-bus power system. Figure (1.5) illustrates the case when both L_{12} and L_{21} are supplied by generator two, that is, $U_{12} = 2$ and $U_{21} = 2$.



It should be noted that the diagonal entries of the loss index matrix, U_{ii} , are set to i . This is purely arbitrary and inconsequential, since, as will be shown later, the diagonal entries of the transaction matrix are allocated zero loss.

2.4 Theory of Loss Allocation

2.4.1 Background

The mathematical model of arbitrary transactions and their relation to the load flow equations is now presented. For simplicity, only the real power-angle component of the load flow equations is considered - in essence assuming that the voltage magnitudes are held constant by sufficient var sources. Thus, the load flow equations for a power system consisting of nb buses are of the form,

$$P_g - P_d = P(\delta) \quad (2.1)$$

where P_g is the nb -vector of real power bus generations; P_d , the nb -vector of real power bus loads; and $P(\delta)$, the nb -vector function relating the net bus injections to the vector of unknown voltage phase angles, δ , including the reference bus.

In the general case, in which losses can be obtained from any generator, and not necessarily the same generator supplying the load contract, the set of all possible bilateral transactions between generator and load buses can be completely represented by three matrices, GD , L , and U . An individual transaction involving buses i, j , and k (where buses i and j represent the generation and load buses participating in the load contract (GD_{ij}), and bus k supplies the contractual losses) can be described by a set of three elements, or the triad, $(GD_{ij}, L_{ij}, U_{ij})$, where

GD_{ij} = power scheduled to be received by a load at bus j from a generator at bus i ,

L_{ij} = the contractual losses attributed to the transaction GD_{ij}

and

U_{ij} = the index of the bus, k , supplying the contractual losses, L_{ij} .

The scheduled contracts, $\{GD_{ij}\}$, are assumed to be specified by the market, that is, these quantities are the given independent variables. The problem of loss allocation is to determine, for a given set of load contracts, $\{GD_{ij}\}$, the corresponding variables $\{L_{ij}\}$, in other words, the matrix L . The role of the loss index matrix, U , is simply to attribute the entries of the loss matrix to the appropriate generator to enable the calculation of the bus generation vector, P_g .

In a practical situation, there does not exist a generator at each bus, nor is each load likely to be involved in transactions with all generators at any given moment. Consequently, the load matrix, GD , the loss matrix, L , and the index matrix, U , will likely be sparse. Nonetheless, in the most general case, these matrices are of dimension $nb \times nb$, where nb is the number of buses in the system.

Assuming that transactions can take place between any pair of buses in a network then the vector of bus loads can be represented as

$$P_d = GD^T e \quad (2.2)$$

where e is a vector with nb elements, each equal to one. Each bus generation can be expressed in the form

$$P_{gi} = \sum_j GD_{ij} + \sum_{U_{kl}=i} L_{kl} \quad (2.3)$$

or in vector form as

$$P_g = GDe + \bar{L} \quad (2.4)$$

where \bar{L} is a vector defined by

$$\bar{L}_i = \sum_{U_k=i} L_{kl} \quad (2.5)$$

\bar{L}_i represents the total amount of generation at bus i dedicated to supplying contractual losses.

The importance of the generalized loss allocation scheme, that is, allowing a consumer to purchase losses from any generator, stems from economic considerations. While it may be true that, prior to the inclusion of losses, it may be more economical for a load at bus j to be supplied by a generator at bus i , this may not be the case after the cost of losses have been added. A method of determining the best choices for loss generation will be discussed in a subsequent chapter on loss allocation. Presently, only the theoretical development of the relevant equations are addressed.

It should be noted that there is a simple relationship between contractual losses and total system losses. The total system losses, P_{loss} , can be expressed in terms of the generation and load vectors or, alternatively, in terms of the loss matrix as

$$\begin{aligned} P_{loss} &= e^T(P_g - P_d) \\ &= e^T L e \end{aligned} \quad (2.6)$$

or, equivalently,

$$P_{loss} = \sum_{i=1}^n \sum_{j=1}^n L_{ij} \quad (2.7)$$

which is the sum of all contractual losses.

2.4.2 Incremental Relations

From the load flow equations (2.1) as well as from (2.2) and (2.4), it follows that for arbitrary infinitesimal changes in the transaction matrices,

$$dP_g - dP_d = [dGD - dGD^T]e + d\bar{L} = \left[\frac{\partial P(\delta)}{\partial \delta} \right] d\delta \quad (2.8)$$

Recall that P_g , P_d , and δ are vectors of length nb and, hence, the Jacobean is a square matrix of dimension $nb \times nb$. Now, it is well-known that the Jacobean matrix, $\frac{\partial P}{\partial \delta}$, is singular and, under normal conditions¹, has rank $nb-1$. As a result, there exists a non-zero nb -dimensional vector α with the property,

$$\alpha^T \left[\frac{\partial P(\delta)}{\partial \delta} \right] = 0 \quad (2.9)$$

which, from (2.8), requires that the incremental transaction matrices should satisfy

$$\alpha^T dP = \alpha^T [(dGD - dGD^T)e + d\bar{L}] = 0 \quad (2.10)$$

¹ One exception being the maximum power transfer conditions when the Jacobian can be of lower rank.

2.4.3 Incremental Loss Allocation Rule

The following general loss allocation rule is proposed for the purpose of determining contractual losses:

An infinitesimal increment in a scheduled load contract, GD_{ij} , results in an infinitesimal increment in its corresponding contractual losses, L_{ij} only. No other contractual loss allocations are affected.

The reasoning behind this rule is that if the scheduled load contract, GD_{ij} , is varied by a small amount, the system operating point is not altered significantly so that only the corresponding loss component is affected. The same is not necessarily true for large increments since such increments would alter the system operating point, thereby influencing the losses attributed to all other transactions as well.

In the context of loss allocation, the unknown variables of interest are the contractual losses, L_{ij} . Assuming contractual losses are supplied by bus k , applying the relation

$$\alpha^T dP = 0 \quad (2.11)$$

in conjunction with the generalized loss allocation rule to each transaction, yields a set of equations relating the contractual losses to their corresponding load contracts, i.e.,

$$\alpha_i dGD_{ij} - \alpha_j dGD_{ij} + \alpha_k dL_{ij} = 0 \quad (2.12)$$

Equation (2.12) can now be solved for the contractual losses as a function of the load contract, that is,

$$dL_{ij} = \frac{(\alpha_j - \alpha_i)}{\alpha_k} dGD_{ij} \quad (2.13)$$

or, equivalently, represented in differential equation form as

$$\frac{dL_{ij}}{dGD_{ij}} = \frac{(\alpha_j - \alpha_i)}{\alpha_k} \quad (2.14)$$

In the case that the contractual loads are functions of time, (2.14) can also be expressed as

$$\frac{dL_{ij}}{dt} = \frac{(\alpha_j - \alpha_i)}{\alpha_k} \frac{dGD_{ij}}{dt} \quad (2.15)$$

or, in matrix form, as

$$\frac{dL}{dt} = H_L(t) \cdot \frac{dGD}{dt} \quad (2.16)$$

where $H_{Lij} = \frac{(\alpha_j - \alpha_i)}{\alpha_k}$ and \cdot denotes element by element multiplication. It should be noted that H_L is a function of α and dependent on the values of L and GD , which can in general be functions of time. Remark that the null space of $\frac{\partial P}{\partial \delta}$ is one dimensional and that ratios of elements of α are unique. Hence, the loss matrix is also unique.

2.4.4 Incremental Transmission Loss Coefficients and the Loss Allocation Rule

The proposed loss allocation rule can also be expressed in terms of the Incremental Transmission Loss (ITL) coefficients. In general, the change in total system losses resulting from incremental variation in net bus injections can be expressed as

$$\sum_1^{nb} dP_i = dP_{loss} \quad (2.17)$$

It is also a well known fact that, given a slack bus, k , the incremental system losses can also be written as

$$dP_{loss} = \sum_1^{nb} \left[\frac{\partial P_{loss}}{\partial P_i} \right]_k dP_i \quad (2.18)$$

where $\left[\frac{\partial P_{loss}}{\partial P_i} \right]_k$ is the sensitivity of the losses with respect to an injection at bus i , with bus k as the slack. Note that the k^{th} sensitivity coefficient is zero. It follows from (2.17) and (2.18) that

$$\sum_1^{nb} \left(1 - \left[\frac{\partial P_{loss}}{\partial P_i} \right]_k \right) dP_i = 0 \quad (2.19)$$

Comparison with equation (2.11) reveals that a valid choice for the elements of the null space vector, α , is

$$\alpha_i = 1 - \left[\frac{\partial P_{loss}}{\partial P_i} \right]_k \quad (2.20)$$

As previously stated, under normal operating conditions, the dimension of the null space of $\left[\frac{\partial P(\delta)}{\partial \delta} \right]$ is one. Therefore, the elements of α are determined within a scaling constant, implying that ratios of elements are unique.

Equation (2.20) is important because it relates the null-space vector α required to solve the loss allocation problem to traditional system parameters, namely, the ITL coefficients. Furthermore, equation (2.19) also permits the calculation of the loss sensitivity with any bus as the slack solely from the knowledge of the sensitivity with respect to one particular slack bus. Expanding equation (2.19) with bus j as the slack bus and dividing by $1 - \left[\frac{\partial P_L}{\partial P_k} \right]_j$ yields the relation

$$\frac{(1 - \left[\frac{\partial P_L}{\partial P_1} \right]_j) \Delta P_1 + \dots + \Delta P_j + \dots + (1 - \left[\frac{\partial P_L}{\partial P_k} \right]_j) \Delta P_k + \dots + (1 - \left[\frac{\partial P_L}{\partial P_{nb}} \right]_j) \Delta P_{nb}}{(1 - \left[\frac{\partial P_L}{\partial P_k} \right]_j)} = 0 \quad (2.21)$$

Following division, the coefficient of ΔP_k is one. Applying equation (2.19) a second time with bus k as the slack also produces an expression (2.22) in which the coefficient of ΔP_k is one.

$$(1 - \left[\frac{\partial P_L}{\partial P_1} \right]_k) \Delta P_1 + \dots + (1 - \left[\frac{\partial P_L}{\partial P_j} \right]_k) \Delta P_j + \dots + \Delta P_k + \dots + (1 - \left[\frac{\partial P_L}{\partial P_{nb}} \right]_k) \Delta P_{nb} = 0 \quad (2.22)$$

Since α is unique within a scaling constant, all other coefficients of equations (2.21) and (2.22) must also be equal. Hence,

$$\frac{(1 - \left[\frac{\partial P_L}{\partial P_i} \right]_j)}{(1 - \left[\frac{\partial P_L}{\partial P_k} \right]_j)} = (1 - \left[\frac{\partial P_L}{\partial P_i} \right]_k) \quad (2.23)$$

2.5 Theory of Line-Loading Allocation

2.5.1 Background

As is the case for loss allocation, a notation must be introduced for the purpose of tracking contractual line flow allocations. For this purpose, the following notation is introduced:

P_{flow} = arbitrary net transmission line flow

P_f = matrix of contractual line flow components, where $P_{f,ij}$ represents the flow component corresponding to the transaction GD_{ij} .

The sum of all the contractual line flow allocations corresponding to a transmission line is equal to the net line flow, P_{flow} . That is,

$$P_{flow} = \sum_{i,j} P_{f,ij} \quad (2.24)$$

As before, it is assumed that voltage magnitudes are fixed at one per unit, so that an arbitrary transmission line flow P_{flow} can be represented as

$$P_{flow} = P_{flow}(\delta) \quad (2.25)$$

where δ is the vector of voltage phase angles, including the reference bus.

2.5.2 Incremental Relations and the Line-Loading Allocation Rule

The following incremental line-loading allocation rule is proposed for the purpose of determining contractual line flow allocations:

An infinitesimal change in a transaction $G D_{ij}$ results in an infinitesimal change its corresponding contractual line flow component, P_{fij} only; no other contractual line flow components are affected.

The reasoning behind this rule is that an infinitesimal change in a transaction results in an insignificant change in the system operating point so that only the corresponding line flow component is modified. This is not true for large changes, since such changes have a drastic affect on the operating point, thereby influencing all contractual line flow allocations.

From (2.8) and (2.25), it follows that

$$\begin{aligned} dP_{flow} &= \left[\frac{\partial P_{flow}(\delta)}{\partial \delta} \right]^T d\delta \\ &= \left[\frac{\partial P_{flow}(\delta)}{\partial \delta} \right]^T \left[\frac{\partial P(\delta)}{\partial \delta} \right]^{-1} (dP_g - dP_d) \end{aligned} \quad (2.26)$$

Defining the vector of flow sensitivity coefficients as

$$\beta^T = \left[\frac{\partial P_{flow}(\delta)}{\partial \delta} \right]^T \left[\frac{\partial P(\delta)}{\partial \delta} \right]^{-1} \quad (2.27)$$

and applying (2.26), in conjunction with the incremental line-loading allocation rule to each transaction, yields

$$dP_{fij} = \beta_i dGD_{ij} - \beta_j dGD_{ij} + \beta_k dL_{ij} \quad (2.28)$$

Substituting (2.13) for dL_{ij} produces the desired relation

$$dP_{fij} = \left[\beta_i - \beta_j + \beta_k \left(\frac{\alpha_j - \alpha_i}{\alpha_k} \right) \right] dGD_{ij} \quad (2.29)$$

Equation (2.29) can be represented in differential equation form as

$$\frac{dP_{fij}}{dGD_{ij}} = \left[\beta_i - \beta_j + \beta_k \left(\frac{\alpha_j - \alpha_i}{\alpha_k} \right) \right] \quad (2.30)$$

or, in the case that the loads are functions of time, as

$$\frac{dP_{fij}}{dt} = \left[\beta_i - \beta_j + \beta_k \left(\frac{\alpha_j - \alpha_i}{\alpha_k} \right) \right] \frac{dGD_{ij}}{dt} \quad (2.31)$$

The corresponding matrix representation of (2.31) is of the form

$$\frac{dP_f}{dt} = H_f * \frac{dGD}{dt} \quad (2.32)$$

where $H_f = \beta_i - \beta_j + \beta_k \left(\frac{\alpha_j - \alpha_i}{\alpha_k} \right)$. Note that the entries of H_f are functions of GD and L, and, therefore, functions of time.

2.5.3 Line-Loading Allocation Using a DC Loadflow Formulation

It is useful to compare the exact line-flow allocation results to those obtained using a DC loadflow formulation, the latter being preferable because it renders network relations linear resulting in constant line flow sensitivities. Unfortunately, the DC loadflow formulation is only valid under the conditions that (i) voltage magnitudes are fixed at one per unit, (ii) line resistances are much smaller than line reactances, and (iii) phase angle differences are small in magnitude.[17] Deviations from these assumptions render the DC loadflow solutions

insufficient and, as a result, unacceptable for allocation purposes.

As previously mentioned, the DC loadflow assumptions result in linear approximations to what are otherwise highly nonlinear network relationships. In particular, the relationship between the vector of net bus injections and the vector of voltage phase angles becomes

$$P = P_g - P_d = (GD - GD^T)e = B\delta \quad (2.33)$$

where B is a real symmetric matrix of rank $(nb-1)$, nb being the number of buses, whose entries are completely defined by the network line susceptances.[17]

It can be easily shown that an arbitrary net flow, P_f , for a line connecting buses i and j can be expressed as

$$P_f = b(\delta_i - \delta_j) \quad (2.34)$$

where b is the susceptance of the line connecting buses i and j . Defining e_i to be a column vector of length nb having zeroes in all entries with the exception of positions i , which has a one, and combining equations (2.33) and (2.34) yields the relation

$$P_f = b(e_i - e_j)^T B^{-1}P \quad (2.35)$$

where B^{-1} is the (left) pseudo inverse of B . Since the DC loadflow approximations result in a lossless network, the vector of net bus injections is completely described by the transaction matrix GD . Therefore, defining e to be a column vector of ones having length nb , and substituting for P provides a relation between the line flow, P_f and the transaction matrix, GD ,

$$P_f = b(e_i - e_j)^T B^{-1}(GD - GD^T)e \quad (2.36)$$

or equivalently,

$$P_f = \sum_{k,l} [b(e_i - e_j)^T B^{-1} e_{kl}] GD_{kl} \quad (2.37)$$

Note that diagonal terms of the transaction matrix, GD , do not influence a line flow, since they are injected and removed at the same bus thereby having no net effect. Clearly then, the sensitivity of an arbitrary line flow, P_f , connecting buses i and j , with respect to a transaction GD_{kl} is

$$\frac{\partial P_f}{\partial GD_{kl}} = b_{ij}(e_i - e_j)^T B^{-1}(e_k - e_l) \quad (2.38)$$

The advantage of the linearity resulting from the application of the DC loadflow approximation is that sensitivities are constant, allowing a market participant to calculate his/her DC loadflow approximation to the line flow allocation independently of any other existing transactions.

2.6 Full Formulation of Loss and Line-Loading Allocation Problem

The full allocation problem is shown here for convenience. To find the matrix of contractual losses, L , and the line flows $\{P_{ij}\}$, solve the system of differential equations

$$\frac{dL}{dt} = H_L(t) \cdot \frac{dGD}{dt} \quad (2.39)$$

where $H_{Lij} = \frac{\alpha_j - \alpha_i}{\alpha_k}$, and

$$\frac{dP_f}{dt} = H_f * \frac{dGD}{dt} \quad (2.40)$$

where $H_{fij} = \beta_i - \beta_j + \beta_k \left(\frac{\alpha_j - \alpha_i}{\alpha_k} \right)$, subject to the relations

$$\alpha^T \left[\frac{\partial P(\delta)}{\partial \delta} \right] = 0 \quad (2.41)$$

and

$$P_g - P_d = (GD - GD^T)e + \bar{L} = P(\delta) \quad (2.42)$$

\bar{L}_i represents the amount of generation at bus i dedicated to supplying contractual losses. In addition, recall that

$$\beta^T = \left[\frac{\partial P_{flow}(\delta)}{\partial \delta} \right]^T \left[\frac{\partial P(\delta)}{\partial \delta} \right]^{-1} \quad (2.43)$$

2.7 Numerical Solution of Allocation Problem

The loss and line-loading allocation problems can be uniquely solved if we assume that the load transaction matrix, GD , follows a one-dimensional path so that $GD=GD(t)$, for $0 \leq t \leq T$, where t is time. The systems of differential equations can be solved numerically using standard numerical algorithms such as the Euler method. It should be noted that the entries of the matrices, H_f and H_L , are determined by the value of the Jacobean null space vector, α ,

and are, consequently, time dependent, since α is a function of the system operating point. Consequently, these matrices must be updated at each step of the numerical integration process. One interesting observation is that the final allocation values at $t=T$ are generally path dependent. In other words, the allocation process is non-conservative and generally cannot be solved from a knowledge of the final value of DG only. Studies also show that paths where the contracts are varied sequentially (say on a first-come-first-served basis) produce substantially different loss allocations depending on the order in which the contracts are processed. The properties of the allocation process will be illustrated and discussed through several examples in the following chapters.

Chapter 3

Loss Allocation

3.1 Introductory Remarks

The loss allocation problem can be solved by assuming that the transaction matrix, GD , follows a one-dimensional path. More specifically, it is assumed that the load can be parametrized by a single scalar parameter, t , possibly representing the time dependence of the system load. The exact loss allocation is found using an Euler method to solve the governing system of differential equations, which is then compared to both approximate and heuristic strategies. Approximate solutions are obtained by using fewer integration steps, while heuristic results are calculated by distributing loss based on the fraction of the total system load represented by each contract. The path dependence of the loss allocation procedure is also investigated and discussed, calculating solutions using linear and nonlinear integration paths. The important topics of approximating or predicting the effect of small load changes on losses are also addressed. The analysis of the loss allocation problem also includes an analysis and discussion of the impact that the choice of loss supplier has on the contractual and total system losses. In conclusion, practical implementations and uses of the loss allocation procedure are suggested and discussed. In this context, the issue of fairness in loss allocation is addressed.

3.2 Preliminary Discussion of Loss Allocation Properties

The advantage of the proposed loss allocation procedure is that, not only does it allocate a corresponding loss component to each bilateral transaction between a generator and load, but that in the process of doing so, it is capable of determining which transactions serve to increase or decrease the total system losses. Those transactions whose net effect (in combination with all the other contracts) is to decrease total system losses are allocated

negative contractual losses, while those which tend to increase total system losses are attributed a positive loss component. Negative loss allocation is undoubtedly a controversial subject and its meaning and interpretation must be addressed.

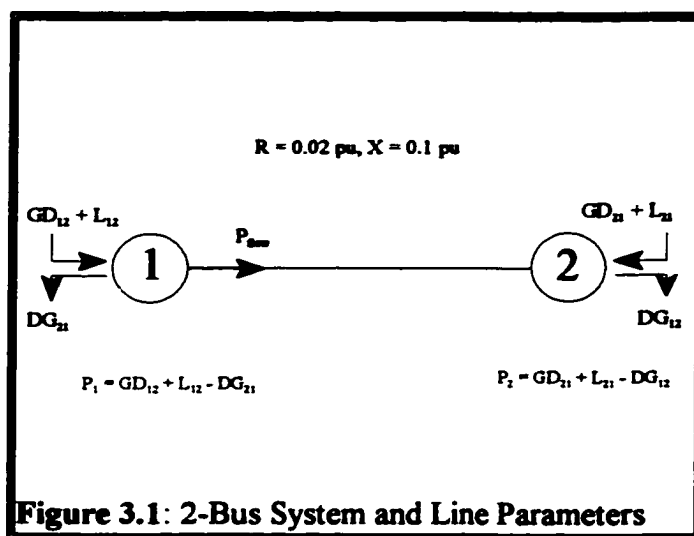
First, it should be noted that the concept of negative loss allocation is not new.[12] It is recognized that given an arbitrary operating point, a change in real power injections can cause the total system losses to increase or decrease. If the incremental injections tend to reduce the current flows on the transmission lines, then total losses will decrease, or, equivalently, if the incremental changes result in a transfer of power which tends to oppose the existing net flows, total losses will be reduced. The primary difference between the negative losses resulting from such incremental changes in real power injections and those calculated using the proposed loss allocation procedure is that the latter determines and allocates the exact amount of positive or negative losses to individual transactions.

A simple two bus example will be used for illustrative purposes. Without loss of generality, the diagonal elements of the load matrix, GD , will be set to zero, since as will be discussed later, the loss allocation procedure attributes zero loss to the diagonal load contracts. The loss index matrix, U , and the experimental value of GD in MW are

$$GD = \begin{bmatrix} 0 & 200 \\ 500 & 0 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \quad (3.1)$$

Three different cases will be addressed using the above matrices. First, it will be assumed that both contracts evolve linearly and simultaneously in time, that is, $GD(t) = GD \cdot t$, $0 \leq t \leq 1$. In the second case, the contractual loads will evolve sequentially. Specifically, the load will evolve linearly on each of two segments: On the first segment, GD_{21} evolves linearly from 0 to 500 with GD_{12} fixed at zero, while on the second, GD_{12} varies linearly from 0 to 200 with GD_{21} fixed at 500. The third case is the reverse of the second; GD_{12} varies from 0 to

200 with GD_{21} fixed at 0, while on the second stage, GD_{21} varies from 0 to 500 with GD_{12} fixed at 200. The two bus system parameters and sign conventions are given in figure (3.1) below. The results are summarized in table (3.1).



As shown in figure (3.1), the convention is that a power flow from bus 1 to bus 2 is considered a positive flow. It is also important to note that all flows are referenced with respect to bus 1. As a result of line losses, numerical power flow results will be slightly different, yet equivalent, if bus 2 is used as the reference. This difference is insignificant for analysis purposes since line losses are small relative to line flows and, therefore, will not affect qualitative interpretations of the data. The values in table (3.1) denoted $(GD_{ij} + L_{ij}, GD_{ij})$ represent contractual generation and load pairs.

Case	Integration	$(GD_{12}+L_{12}, GD_{12})$	$(GD_{21}+L_{21}, GD_{21})$	Losses	P_{flow} Stage 1	Final P_{flow}
1	simultaneous	(187.1,200)	(535.4,500)	22.5	-----	-312.9
2	sequential GD ₂₁ first	(162.3,200)	(564.2,500)	26.5	-500.0	-337.7
3	sequential GD ₁₂ first	(208.8,200)	(510.5,500)	19.3	208.8	-291.2

Note: All results in MW

It should be noted that the solutions to the loss allocation problem are not optimal; the losses have not been minimized. A traditional load flow, with voltage magnitudes fixed at one per unit and bus 1 as the slack bus, yields losses totaling 18.0 MW, clearly showing that there are alternative possibilities for distributing generation which can reduce the real power losses for the load matrix corresponding to the two bus system given above. In fact, in the case that there is a generator at each bus, a zero loss generation pattern can be obtained simply by setting P_{gi} equal to P_{di} . Moreover, the path of integration has a profound effect on real power losses. Results depend upon whether the contracts evolve simultaneously or sequentially, the exact sequence also affecting the line losses. Note that the extremely high losses experienced in case two is due to the fact that the load, $GD_{21} = 500$ MW, on the first stage of the sequential integration process is rather large; that is, the line is heavily loaded, resulting in substantial real power losses.

At this point, an explanation regarding the signs of the loss allocation for individual bilateral transactions is required. As stated earlier, the sign of the loss allocation is dependent on whether a transaction tends to increase or decrease the net line flow. For the two bus example, the line flow is determined by the values of two transactions, GD_{12} and GD_{21} . Clearly, when integrated simultaneously, the direction of the net line flow is determined by

the magnitude of the contracts. For example, if GD_{21} is larger, as is the situation in case 1, the final flow will be directed from bus 2 to bus 1. This result can be understood by considering the integration procedure. For every incremental arc length along the path of integration, the corresponding change in GD_{21} is larger than that of GD_{12} . Consequently, at each step of the integration process, the line flow directed from bus 2 to bus 1 increases in magnitude. In this context, the overall effect of GD_{21} is to increase the magnitude of the flow, while GD_{12} tends to reduce it. Therefore, at each step of the integration procedure, GD_{12} accumulates negative loss, while GD_{21} accumulates positive loss.

Two different scenarios are possible in the case of sequential integration. Cases 2 and 3 in table (3.1) illustrate each case. In the second case, GD_{21} is integrated first, causing a net flow of 500 MW directed from bus 2 to bus 1 at the end of the first stage. Since GD_{12} tends to push power from bus 1 to bus 2, it reduces the flow from bus 2 to bus 1 and consequently receives negative loss allocation. Case 3 depicts a similar integration process. The fact that both transactions receive positive losses is due to the magnitude of GD_{21} ; it is so large that the direction of the net line flow at the conclusion of the integration process is opposite that obtained following the first stage. More specifically, the load contract GD_{21} causes a flow component directed from bus 2 to bus 1. Initially, it opposes the net flow, thereby accumulating negative losses. Eventually, the direction of the net line flow is reversed, following which GD_{21} accumulates positive losses for generating flow components which tend to increase the magnitude of the net line flow. Note that the reversal of the direction of the net line flow on the second stage of the integration process is not enough to determine the signs of the loss allocations. That is, the corresponding allocations could still have been of opposite sign had the contractual load, GD_{21} , been smaller in magnitude.

Note that the above explanation regarding a two bus system is solely for the purpose of understanding the meaning of negative loss allocation and its dependence on the path of integration. These issues are not as clear for larger systems, since the line flows are the result of several transactions, making it more difficult to predict the sign of the loss allocation a

priori. These issues will be addressed in a subsequent section.

To summarize, negative loss allocation simply implies that the net effect of a transaction (in combination with all other transactions) is to reduce real power flow. One of the goals of this thesis is to address this issue, which although not new, has not received due consideration.

The first step in the analysis process is to examine the fundamental properties of the loss allocation procedure. To this end, loss allocation is performed using a 5-bus power system for both linear and non-linear integration paths, the purpose of the latter being to understand the effect of loads which may arrive in batches, rather than simultaneously, or in other words to study the effects of loads which can be to a large extent random. Heuristic and approximate allocations are also investigated for the purpose of comparison to the exact results obtained using the proposed theory. The first portion of the analysis assumes that both load and loss contracts are supplied at the same bus.

3.3 Formulation of Loss Allocation Problem for a Linear Contract Path

The numerical solutions for the linear contract path are obtained using an Euler numerical integration technique to solve the system

$$\frac{dL}{dt} = H_L(t) \cdot \frac{dGD}{dt}, \quad GD(t) = GDt, \quad t \in [0,1] \quad (3.2)$$

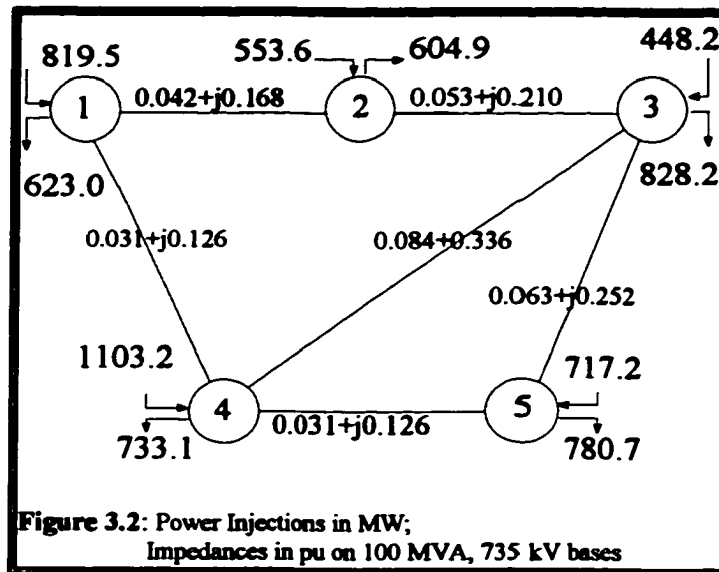
where $H_{L_{ij}} = \frac{\alpha_j - \alpha_i}{\alpha_k}$, subject to the relation

$$\alpha^T \left[\frac{\partial P(\delta)}{\partial \delta} \right] = 0 \quad (3.3)$$

and the load flow equations in terms of the transaction matrix, DG, and loss vector, \bar{L} ,

$$P_g - P_d = (GD - GD^T)e + \bar{L} = P(\delta) \quad (3.4)$$

Note that the time dependence of the matrix, H_t , is merely shown to indicate the fact that it is a function of the operating point, and must consequently be updated at each step of the integration process.



3.4 Simultaneous and Linear Integration Path Results

The 5-bus system shown in figure (3.2) is used to illustrate the concepts presented here. It is assumed that the scheduled contracts at the loads, in MW, and the loss index matrix are given by the matrices GD and L , given as

$$GD = \begin{bmatrix} 65.7 & 14.1 & 203.7 & 203.8 & 280.4 \\ 115.1 & 155.8 & 249.3 & 10.4 & 16.1 \\ 158.9 & 201.3 & 2.3 & 115.0 & 20.0 \\ 125.3 & 206.0 & 176.7 & 279.1 & 253.9 \\ 158.1 & 27.6 & 196.2 & 124.8 & 210.4 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (3.5)$$

Recall that the current loss index matrix implies that the contractual losses are supplied by the same generator that is supplying the corresponding load. The first result, (3.6), shows the MW Loss matrix, L ,

$$L = \begin{bmatrix} 0.0 & 1.2 & 32.3 & -0.1 & 18.4 \\ -8.6 & 0.0 & 16.7 & -0.8 & -0.3 \\ -20.6 & -12.3 & 0.0 & -14.9 & -1.5 \\ 0.0 & 17.3 & 28.1 & 0.0 & 16.7 \\ -9.5 & 0.5 & 16.7 & -7.5 & 0.0 \end{bmatrix} \quad (3.6)$$

It is first observed that the diagonal terms of L are zero because contracts within the same bus do not affect the transmission losses. Such transactions result in zero net change in power injection. Certain contracts, such as GD_{13} and GD_{43} , have the highest losses allocated to them while contracts GD_{14} and GD_{52} are allocated relatively little loss. It is difficult to interpret these results solely on the basis of network impedances. For example, the reasons may be that line 3-4 has a relatively high resistance while buses 1 and 3 do not have a direct path, but such interpretations cannot be generalized. The negative loss allocation is a surprising and, at first glance, counterintuitive phenomenon. The fact that contract GD_{31} is allocated negative loss, for example, simply indicates that its principal global impact (in combination with the other contracts) is to reduce overall system losses or, equivalently, that GD_{31} produces system flows which tend to oppose the net line flows, effectively decreasing real power losses. This allocation scheme therefore assigns a generation component at bus 3 of

138.3, that is, 158.9 – 20.6 MW, in order to meet the corresponding contract at bus 1 of 159 MW! The question of which party or parties should benefit from such negative loss allocation will be addressed later. Furthermore, the opposite signs of L_{13} and L_{31} exhibit a property of the simultaneous and linear integration loss allocation procedure: *The losses allocated to a pair of contracts between any two buses, i and j , namely L_{ij} and L_{ji} , always have opposing signs.* This result is intuitively consistent, since such pairs of contracts produce opposing network flows. Thus, the corresponding loss allocations are of opposite sign at each integration step.

It is also informative to note that close observation of the loss matrix reveals a trend in the sign and magnitude of the loss allocation components. The loss component allocated to a bilateral transaction between a generator at bus i and a load at bus j is positive if $P_i - P_j \gg 0$ and negative if $P_i - P_j \ll 0$. That is, the transaction is allocated a negative loss component if it opposes the general trend of power flow from bus i to j . The results are more ambiguous for cases in which $P_i - P_j \approx 0$ as is the case for the bilateral transaction, GD_{52} , which is allocated positive rather than negative loss, in apparent violation of the rule stated above.

The above rule helps to explain why all the load contracts associated with some buses receive solely positive or negative losses. For example, $P_i - P_3 \gg 0 \quad \forall i \neq 3$; therefore, all bilateral transactions of the form GD_{i3} , that is, involving a generator at bus i and a load at bus 3 would be expected to be allocated positive loss, as in indeed the case, and which can be verified by investigating the third column of the loss matrix. In contrast, and by the same argument, all transactions of the form GD_{i4} would be expected to be allocated negative loss.

3.5 Approximate and Heuristic Loss Allocation

Two alternative methods for loss allocation have been examined and compared to the exact method presented above. One is an approximation based on a two-step Euler integration of the defining differential equations, while the second method uses a heuristic where losses are allocated in proportion to the magnitude of the contract. In the latter case, the loss allocation

is always positive; that is, everyone is responsible for some fraction of the system losses, and no contract gets rewarded. Specifically, the heuristic losses were obtained by letting

$$\begin{aligned} L_{ij} &= \rho GD_{ij} \quad i \neq j \\ L_{ii} &= 0 \end{aligned} \quad (3.7)$$

Since losses are non-negative, ρ is greater than or equal to zero. The diagonal elements, L_{ii} , were set equal to the corresponding loads since the proposed loss allocation procedure allocates zero loss to such transactions. Equation (3.8) shows the L matrices for the approximate Euler and the heuristic method. These matrices are denoted by L_{eu2} and L_{heu} . Note that the loss index matrix, U , shown in equation (3.5) was used for the two-step Euler approximation.

$$L_{eu2} = \begin{bmatrix} 0.0 & 0.7 & 15.6 & 2.0 & 10.7 \\ -2.6 & 0 & 9.6 & -0.2 & 0.0 \\ -7.5 & -3.1 & 0 & -5.6 & -0.5 \\ 1.5 & 9.9 & 13.7 & 0 & 10.0 \\ -2.4 & 0.5 & 9.2 & -2.0 & 0 \end{bmatrix} \quad L_{heu} = \begin{bmatrix} 0.0 & 0.3 & 3.6 & 3.6 & 5.0 \\ 2.0 & 0 & 4.4 & 0.2 & 0.3 \\ 2.8 & 3.6 & 0 & 2.0 & 0.4 \\ 2.2 & 3.7 & 3.1 & 0 & 4.5 \\ 2.8 & 0.5 & 3.5 & 2.2 & 0 \end{bmatrix} \quad (3.8)$$

Note that the results obtained using an approximate two-step Euler procedure had to be modified to make them loadflow consistent. This was accomplished by applying equation (3.7) at each intermediate step. Comparing the two step Euler to the exact solution (3.6), it can be seen that the results have the correct trend but exhibit significant error. This property can be exploited to reduce the computation time, especially for larger systems; reasonably accurate and acceptable results can be obtained using relatively few integration steps. In contrast, the heuristic allocation method shows no resemblance whatsoever to the exact solution, an expected result considering that, with the exception of the loadflow equations, this scheme ignores all network relationships, and is therefore incapable of determining a fair loss allocation.

The above results were obtained by assuming that all contracts approach their final value uniformly or, in other words, that $GD(t)$ evolves linearly in time. However, in practice, contracts may materialize sequentially, individually or in batches, thus resulting in varying paths from one level of GD to another. An investigation of such phenomena follows.

3.6 Path Dependent Loss Allocation

To examine the nature of the path dependence of the loss allocation scheme, the following scenario was implemented. Rather than allowing all contracts to vary linearly for t ranging from 0 to 1, the contracts were restricted to a predetermined nonlinear path, composed of two linear sub-paths. The sub-paths were determined by decomposing GD in equation (3.5) into the sum of two contract matrices, GDA and GDB. GDA is identical to GD, with the exception that GDA_{32} is zero. For the second linear sub-path, all entries of GDB are zero with the exception that GDB_{32} is equal to GD_{32} . The contractual losses were assumed to be generated by the same bus supplying the corresponding load contract; that is, the loss index matrix is as shown in equation (3.5). This scheme, representing a simplified batch arrival, non-linear scheme, is sufficient for the purpose of determining properties of the non-linear allocation procedure.

The loss matrices corresponding to the end of the first linear sub-path and the completed path, L_A and L_B , are calculated to be

$$L_A = \begin{bmatrix} 0.0 & 0.7 & 51.5 & 5.9 & 32.5 \\ -5.5 & 0.0 & 46.7 & -0.2 & 1.0 \\ -29.4 & 0.0 & 0.0 & -18.8 & -2.0 \\ -3.5 & 4.4 & 37.9 & 0.0 & 21.1 \\ -15.7 & -1.5 & 22.9 & -9.3 & 0.0 \end{bmatrix} \quad L_B = \begin{bmatrix} 0.0 & 0.7 & 51.5 & 5.9 & 32.5 \\ -5.5 & 0.0 & 46.7 & -0.2 & 1.0 \\ -29.4 & -51.4 & 0.0 & -18.8 & -2.0 \\ -3.5 & 4.4 & 37.9 & 0.0 & 21.1 \\ -15.7 & -1.5 & 22.9 & -9.3 & 0.0 \end{bmatrix} \quad (3.9)$$

First, although the final load matrix, GD, is the same for both the linear and nonlinear contract paths, the final operating point, defined by the combined values of GD and L_B , is different.

Secondly, comparison of the loss matrices in equations (3.6) and (3.9), reveals that while most of the signs are identical, the magnitudes differ significantly. For example, the transaction, GD_{32} has been further rewarded as a result of the nonlinear contract path. While L_{32} was -12.3 MW in the linear case, it is now -51.4 MW.

Furthermore, summing the elements of the loss matrices reveals that the total system loss decreased from 138.7MW to 87.2MW as a result of the addition of contract GD_{32} on the second stage of the nonlinear path. Inspection of (3.9) reveals that all the benefit derived from this decrease in system loss was attributed to GD_{32} . Specifically, the benefit allocated to GD_{32} , can be easily verified to be equal to the change in system loss occurring on the second stage of the nonlinear path. This result reflects an important characteristic of the loss allocation procedure: *Loss allocation is changed only for those contracts which have been modified.* This result is consistent with equation (3.2), which states that L_{ij} is constant whenever GD_{ij} is constant. An unfortunate consequence of this property is that in the event that relatively few contracts are modified, using a nonlinear approach results in all of the benefit or penalty being allocated to few parties. It can be argued that this is unfair, since any reduction or increase in loss is a complex nonlinear function of all contracts. Experimental results show that loss is more evenly and, arguably, more fairly distributed using a linear contract path and reintegrating starting at $t=0$. This issue will be revisited later.

As previously stated and shown for a two bus example in table (3.1), the loss allocation results are not necessarily optimal. In fact, the final allocation is very much dependent on the choice of integration path. To further illustrate this fact, results were calculated for an alternative integration path obtained by reversing the order of the linear subpaths. That is, all entries of GDA were zero with the exception of GDA_{32} , which was set equal to GD_{32} , while GDB was identical to GD, with the exception of GDB_{32} , which was set to zero. The corresponding allocation results are

$$L_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 17.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad L_B = \begin{bmatrix} 0 & 1.6 & 15.0 & -6.9 & 3.8 \\ -11.6 & 0 & -9.3 & -1.4 & -1.4 \\ -9.1 & 17.4 & 0 & -10.3 & -1.0 \\ 4.4 & 31.2 & 19.5 & 0 & 12.3 \\ -1.7 & 2.7 & 11.1 & -5.6 & 0 \end{bmatrix} \quad (3.10)$$

The total losses for this case, obtained by summing all the elements of L_B , are 60.7 MW. Note that slight inaccuracies may result from round off errors. Again, the loss allocation is unchanged for constant contracts; $L_{A_{32}}$ and $L_{B_{32}}$ are equal, since no change occurred in the corresponding load contract on the second stage.

As mentioned earlier, the loss allocations corresponding to pairs of contracts between two buses i and j , namely L_{ij} and L_{ji} , are always of opposite sign for linear integration paths; this is not the case for nonlinear paths. Recall that the loss allocations for the contracts which are unchanged remain constant and note that the sign of a loss allocation is largely dependent on the direction of the net line flows. If the addition of a contract is such that it causes the direction of one or many line flows to reverse, then it is possible that L_{ij} and L_{ji} will have the same sign for some buses i and j . An example is presented as proof of this statement. The load matrix, GD , and the resulting loss matrix, L , are shown in equation (3.11). The nonlinear path is composed of two sequential linear subpaths. As before, GD is decomposed into the sum of two matrices, GDA and GDB . GDA is identical to GD , with the exception that GDA_{42} is zero, while the only non-zero entry of GDB is GDB_{42} which is equal to GD_{42} .

$$GD = \begin{bmatrix} 80 & 20 & 240 & 240 & 360 \\ 120 & 200 & 320 & 40 & 20 \\ 200 & 280 & 40 & 160 & 40 \\ 160 & 280 & 200 & 360 & 320 \\ 200 & 40 & 260 & 160 & 280 \end{bmatrix} \quad L = \begin{bmatrix} 0.0 & 0.5 & 44.9 & 15.9 & 46.8 \\ -2.7 & 0.0 & 50.7 & 1.7 & 2.1 \\ -29.4 & -36.1 & 0.0 & -15.2 & -1.8 \\ -9.7 & 14.4 & 22.0 & 0.0 & 18.6 \\ -21.9 & -3.6 & 12.5 & -8.6 & 0.0 \end{bmatrix} \quad (3.11)$$

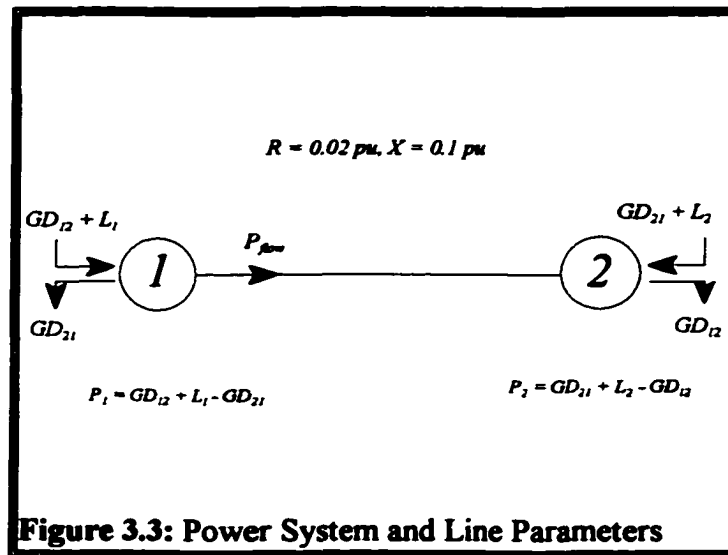
Inspection of L reveals that L_{42} and L_{24} are both positive or, equivalently, that both are penalized, an occurrence which is only possible using a nonlinear integration path. Similarly, it is also possible that the losses are both negative.

Furthermore, the change in system loss occurring as a consequence of the addition of any contracts can be positive or negative, depending on the actual value of GDA and GDB. Finally, the total loss obtained for a nonlinear integration path can be either higher or lower than that for a linear path. The numerical results shown here are not meant to reflect general trends in these regards.

So far, all examples have assumed that contractual losses were supplied at the same bus generating the corresponding load contract; that is, the loss index matrix was assumed to be of the form shown in equation (3.5). In general, it is possible for a load to have its contractual losses supplied by any generator, whether it be for technical or financial reasons. Hence, this more general scenario must be investigated, and is therefore the next topic to be addressed.

3.7 Generalized Loss Allocation

The primary aspect of a market which promotes economic efficiency is consumer choice. In a competitive environment, a load consumer, subject to the condition that network security and reliability remain inviolated, has the right to purchase power from any generator. This privilege should not only apply to the load transaction itself, as defined by GD_{ij} , but also to the loss transaction, L_{ij} , since it is possible that the amount or cost of losses can be reduced if purchased from a generator other than that supplying the contract.



The two bus system shown in figure (3.3) is used to investigate the nature of the generalized loss allocation procedure and to show the effect of the choice of generator on total and contractual losses. Recall that \bar{L}_i represents the amount of generation at bus i dedicated to transmission losses. Four different cases are investigated using the MW load matrix,

$$GD = \begin{bmatrix} 0 & 500 \\ 200 & 0 \end{bmatrix} \quad (3.12)$$

The first case shows the loss allocation obtained using the traditional scheme in which both the losses and the load contract are supplied by the same generator. Cases two and three address the situation in which all the losses are supplied by buses one and two, respectively. The last case considers the scenario where a load buys its losses from local generation, that is, the generator located at the same bus. The results are shown in table (3.2). Note that all contracts are assumed to evolve simultaneously and linearly in time.

Case	Total Losses (MW)	L_{12} (MW)	L_{21} (MW)
Losses/contract from same generator	22.5	36.4	-12.9
All losses from bus 1	20.6	34.3	-13.7
All losses from bus 2	18.0	30.1	-12.0
Contractual Losses supplied locally	16.6	29.2	-12.7

Note that the contract involving the generator at bus one and load at bus two is by far the larger of the two. Consequently, the net line flow is always from bus one to bus two. Furthermore, bus one is a net generator bus, while bus two is a net load bus. These facts, in conjunction with the previous discussion of negative loss allocation in sections 3.2 and 3.3, reveal that

- (1) injecting power at node 1 or sinking power at node 2 will tend to increase net line flow from bus 1 to 2, thereby resulting in higher losses.
- (2) injecting power at node 2 or sinking power at node 1 will tend to decrease the net line flow from bus 1 to 2, thereby decreasing losses.

The above properties facilitate a qualitative understanding of the loss allocation procedure and the effect of individual bilateral transactions on the total system losses. Recall that it is a fact that L_{12} is positive and L_{21} is negative for the simultaneous integration procedure. Properties (1) and (2), therefore, imply that losses will be minimized if the load at bus 2 buys its (positive) losses, L_{12} , from bus 2 and the load at bus 1 purchases its (negative) losses, L_{21} , from bus 1. This is so because injecting L_{12} will create a flow component from bus 2 to 1. Injecting the negative loss component L_{21} at bus 1, or equivalently, sinking $-L_{21}$, reduces the

net injection at bus 1, thereby reducing the net flow from 1 to 2. Since the individual effects of each loss component is to reduce the net line flow, this scenario is the best as far as the total system losses is concerned. The total loss for this case, 16.6 MW, is indeed the lowest of the four cases. Unfortunately, similar interpretations do not necessarily apply to individual contractual loss components, since their exact values are determined by the exact values of the entries of α , as defined in equation (3.2). That is, while a scenario may reduce the total losses, it may not be most beneficial for all parties, as testified by the results shown in table (3.2).

The application of the generalized allocation scheme will be addressed in the context of the 5-bus example shown in section 3.3. Recall that the transactions of the form GD_{4i} , that is all transactions involving a generator at bus four, were allocated substantial positive loss components as a result of the fact that $P_4 - P_k \gg 0$, for $k \neq 4$. It can be concluded from the previous discussion of generalized loss allocation that there exists a correlation between magnitude of contractual losses and the difference in net injections at the relevant buses. Consequently, it is wiser for loads purchasing power from bus 4 to obtain losses from local generation, thereby reducing the magnitude of their corresponding contractual losses. The loss index matrix corresponding to this scenario is

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (3.13)$$

The contractual losses obtained using a linear and simultaneous integration procedure are presented in equation (3.14).

$$L = \begin{bmatrix} 0 & 1.1 & 30.6 & 0.7 & 17.8 \\ -8.0 & 0 & 16.4 & -0.7 & -0.2 \\ -19.8 & -12.1 & 0 & -14.0 & -1.4 \\ -0.4 & 13.7 & 21.5 & 0 & 14.0 \\ -9.2 & 0.4 & 15.7 & -6.9 & 0 \end{bmatrix} \quad (3.14)$$

Clearly, the positive losses corresponding to transactions of the form GD_{4i} have decreased substantially. It should be noted that, for similar reasons, the magnitudes of negative contractual losses for bilateral transactions, GD_{i4} , that is, involving a load at bus 4, have also decreased.

3.8 Loss Allocation and Choice of Supplier

3.8.1 Introduction

The loss allocation results shown in section (3.7) indicate that a consumer should be prudent when choosing which generator or generators to contract for the purpose of supplying the power for the load and the corresponding contractual losses. Firstly, it should be noted that the minimum loss solution may not be the most desirable; the consumer's main concern is cost. While one generator may provide the most economical pre-loss price, the total cost to the consumer following the addition of the cost of losses may not represent the consumer's best option.

Fortunately, the consumer need not be completely ignorant of the effect of his contract on losses. The results shown in table (3.2) and the corresponding discussion in sections (3.6) and (3.7) reveal the following basic rules which should be utilized by the consumer:

- (1) a consumer located at bus j should attempt to purchase power from bus i such that $P_i - P_j \ll 0$, resulting in negative contractual losses.
- (2) contractual losses can be approximated using the entries of the loss sensitivity

matrix, H_L .

As (1) was already discussed in the context of generalized loss allocation, the topic of approximating contractual losses will now be addressed.

3.8.2 Approximating Contractual Losses

The advantage of the Euler numerical integration process is that it allows a simple calculation of the change in contractual loss resulting from an infinitesimal variation in the corresponding load component. The Euler algorithm can also be used to approximate larger changes of the order of five percent with reasonable accuracy using the linearized approximation,

$$\Delta L_{app} = H_L \cdot \Delta GD \quad (3.15)$$

where H_L is the loss sensitivity matrix determined by the current operating point. The prediction procedure was compared to the exact solution obtained using a two-stage integration process. Equation (3.16) shows the first and second stage load matrices, GDA and GDB , in MW. Recall that the loads evolve linearly on each stage, that is, $GDA(t) = GDA t$, $0 \leq t \leq 1$ and $GDB(t) = GDB t$, $0 \leq t \leq 1$. The loss index matrix was chosen to be as shown in equation (3.5), that is, contractual losses were supplied by the same bus generating the load contract. Note that the final operating point achieved at the conclusion of the first stage, determined by (LA, GDA) , defines the initial conditions for the second stage as well as the base operating point for the prediction process.

$$GDA = \begin{bmatrix} 80 & 20 & 240 & 240 & 360 \\ 120 & 200 & 320 & 40 & 20 \\ 200 & 280 & 40 & 160 & 40 \\ 160 & 280 & 200 & 360 & 320 \\ 200 & 40 & 260 & 160 & 280 \end{bmatrix} \quad GDB = \begin{bmatrix} 24.32 & 0 & 0 & 0 & 0 \\ 36.47 & 0 & 0 & 0 & 0 \\ 60.79 & 0 & 0 & 0 & 0 \\ 48.63 & 0 & 0 & 0 & 0 \\ 60.79 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.16)$$

The second stage load, GDB, represents a five percent increase in total system load. The exact value of the loss matrix, LA , corresponding to the first stage in addition to the exact and predicted variation in generation, ΔL_{exact} and ΔL_{app} , resulting from the five percent increase in load are

$$LA = \begin{bmatrix} 0 & 2.3 & 44.7 & -1.1 & 26.9 \\ -12.0 & 0 & 19.1 & -4.2 & -0.7 \\ -29.4 & -15.3 & 0 & -24.1 & -3.5 \\ 0.7 & 34.2 & 38.4 & 0 & 25.6 \\ -13.5 & 1.5 & 26.1 & -11.5 & 0 \end{bmatrix} \quad (3.17)$$

$$\Delta L_{exact} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -6.9 & 0 & 0 & 0 & 0 \\ -16.8 & 0 & 0 & 0 & 0 \\ 2.7 & 0 & 0 & 0 & 0 \\ -5.7 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \Delta L_{app} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -5.6 & 0 & 0 & 0 & 0 \\ -13.9 & 0 & 0 & 0 & 0 \\ 4.9 & 0 & 0 & 0 & 0 \\ -2.0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.18)$$

The results obtained using a first order approximation to predict the impact of GDB are as shown in equation (3.18). Recall, that the integration process only modifies contracts which have undergone a change; therefore, only the first column of the generation matrix experiences a variation. The errors become even more pronounced as the step size, ΔGD , increases. Clearly, the error in approximation can be relatively large when the magnitude of the contractual losses is near zero. Consequently, care should be taken in making such predictions.

Note that the approximation procedure can also be applied to the generalized loss allocation scheme where contractual losses can be supplied by an arbitrary bus.

3.9 Practical Consideration in Loss Allocation

The loss allocation procedure presents two issues which must be addressed: First, a practical yet physically sound method of implementation must be achieved and secondly, the issue of fairness must also be addressed.

It should be noted that the author is not necessarily suggesting that the loss allocation procedure be used for the purpose of real time loss allocation. The author is well aware that the algorithm can be time consuming. Two alternatives are proposed as methods of circumventing this difficulty:

(1) Approximate solutions can be obtained using relatively few steps, thereby allowing the ISO to dictate exactly what generation must be supplied to satisfy the market demand, GD.

(2) In the advent that loss compensation is an ancillary service and open to competition, the loss allocation algorithm can be implemented periodically, off-line, so that losses can be distributed based on percentages dictated by the loss matrix, L . Although not exact, such a procedure would at least retain the complex nonlinear relationships that exist among the various transactions, an accomplishment which is seemingly impossible using heuristic techniques.

The loss allocation scheme also poses an important question: Is it fair to reward (negative loss allocation) or penalize (positive loss allocation) any particular party to an extreme degree? Perhaps, it would be more equitable to treat opposing pairs of contracts (when possible), namely, GD_{ij} and $GD_{j\bar{i}}$. As mentioned earlier the loss allocations for such pairs is always of opposite sign, suggesting that it may not be unreasonable to divide the sum of the loss allocations for such contracts, thereby dividing any benefit or penalty. Of course, the application of such a scheme demands further study regarding the interdependence of all transactions, since it is in essence a compromise between the exact algorithm and heuristics

which could ultimately defeat the purpose for which the allocation scheme was developed in the first place.

3.10 Summary and Conclusions

The main results presented in this chapter are based on the notion that an infinitesimal increment in a bus-to-bus transaction at the receiving end affects only the value of its corresponding contractual loss component. Furthermore, for an infinitesimal variation of a transaction at the receiving end, it is always possible to exactly calculate the corresponding real power loss. Numerical examples illustrate the developed theory including a comparison with more practical approximate methods.

Briefly, experimental results indicate:

- (1) The loss allocation procedure is path dependent.
- (2) Reintegrating all contracts from $t = 0$ may be more equitable, as it distributes any penalty or benefit that may result from the introduction of any new transactions.
- (3) It is possible to obtain an approximate and adequate loss allocation using relatively few integration steps, allowing participants to calculate a simple and rapid approximate allocation solution.
- (4) Heuristic allocation schemes are inaccurate since they cannot possibly reflect the complex nonlinear relationships among transactions.
- (5) The loss allocation algorithm can be used for dispatching or for the purpose of distributing loss in the case that loss replacement is solely an ancillary service.
- (6) There are measures that consumers can take to minimize penalty (positive

loss) or maximize benefit (negative loss), the most important consideration being that a participant should attempt to purchase power from load buses.

The results presented in this chapter are not meant to be exhaustive, but to illustrate the fundamental properties of the loss allocation procedure. There is still much to be discovered and understood concerning the loss allocation process. Further research is therefore required.

Chapter 4

Flow Allocation

4.1 Introductory Remarks

With the advent of competition in the power industry it has been proposed that competing suppliers and consumers be charged for the use of transmission lines. Ideally, this goal requires a method for determining the contribution of different parties to each net line flow, a method which currently does not exist. The prevailing flow allocation schemes were and are still based on the nonphysical notion of *contract paths*, a method which has been proven to be inaccurate, unfair, and unacceptable, since the power transferred as a result of a transaction does not follow a predefined path, but rather spreads throughout the network according to the laws of nature, namely those of Kirchhoff and Ohm. The use of non-scientific *contract paths* is unrealistic and frequently forces transmission providers to curtail legitimate transactions to accommodate flows of unknown origin.

The flow allocation solution proposed in the present thesis is based on the notion that the incremental line flow resulting from an infinitesimal variation in a load contract can be computed exactly. This leads to a system consisting of both algebraic and differential equations.

The exact flow allocation solution is presented for several interesting examples, and is solved by assuming that the transaction matrix, GD , follows a one-dimensional path. More specifically, it is assumed that the load can be parametrized by a single scalar parameter, t , possibly representing the time dependence of the system load.

The exact flow allocation, found using an Euler method to solve the governing system of differential equations, is compared to approximate solutions obtained using fewer integration steps and to the heuristic *contract path* allocation scheme. The validity of the DC loadflow

solution in determining line flow allocations is also investigated. The issue of path dependence of the flow allocation procedure is also analysed and discussed, since system load is not exactly predictable and, therefore, does not necessarily evolve linearly in time. In this context, random fluctuations in demand and nonlinear time evolution of loads are addressed by calculating solutions using linear and nonlinear integration paths. Lastly, practical implementations and uses of the flow allocation procedure, particularly for congestion management, are suggested and discussed.

4.2 Preliminary Discussion of Line-Loading Allocation Procedure

Since the nature of the loss allocation results was discussed in detail in the previous chapter, the introduction to the line-loading problem will be kept brief. First, it should be noted that the development of the loss and line-loading allocation procedures are closely linked, and their solutions, therefore, exhibit similar properties. In addition, as was the case for loss allocation, there are negative line-flow allocations, which simply imply that the net effect of a transaction (in combination with all other transactions) is to produce a flow component which opposes the direction of the net line flow. Results should otherwise not present any difficulty and are, therefore, now presented beginning with simultaneous and linear integration path results.

4.3 Flow Allocation for a Linear Contract Path

The numerical solutions for a linear contract path are obtained using an Euler numerical integration technique to solve the system,

$$\frac{dP_f}{dt} = H_f(t) \cdot \frac{dGD}{dt}, \quad GD(t) = GDt, \quad t \in [0,1] \quad (4.1)$$

where $H_{fj} = \beta_i - \beta_j + \beta_k \frac{\alpha_j - \alpha_i}{\alpha_k}$, where k is the bus supplying the contractual losses for

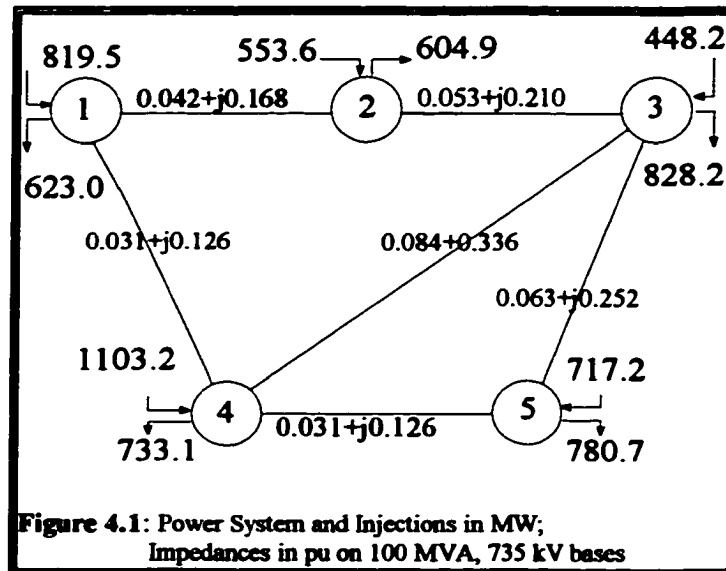
the load contract DG_{ij} , subject to the relation,

$$\alpha^T \left[\frac{\partial P(\delta)}{\partial \delta} \right] = 0 \quad (4.2)$$

and the load flow equations,

$$P_g - P_d = (GD - GD^T)e + d\bar{L} = P(\delta) \quad (4.3)$$

where GD is the known matrix of loads and \bar{L} is the loss vector, \bar{L}_i representing the component of the net injection, P_i , dedicated to contractual losses. Note that the time dependence of the matrix, H_p , is shown to indicate the fact that it is a function of the operating point, and must consequently be updated at each step of the integration process.



4.4 Simultaneous and Linear Integration Path Results

The 5-bus system shown in figure (4.1) is used to illustrate the concepts presented here. The final bus generations, P_g , and loads, P_L , are also shown for convenience. The scheduled contracts at the loads, in MW, and the loss index matrix, are assumed to be given by the matrices,

$$GD = \begin{bmatrix} 65.7 & 14.1 & 203.7 & 203.8 & 280.4 \\ 115.1 & 155.8 & 249.3 & 10.4 & 16.1 \\ 158.9 & 201.3 & 2.3 & 115.0 & 20.0 \\ 125.3 & 206.0 & 176.7 & 279.1 & 253.9 \\ 158.1 & 27.6 & 196.2 & 124.8 & 210.4 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (4.4)$$

Note that this is the same transaction matrix and power system used for loss allocation purposes. (equation (3.5), figure (3.2)) Recall that the loss index matrix in (4.4) implies that the a contract, GD_{ij} , and its corresponding contractual losses are supplied by the same bus. The exact line flows, obtained using an Euler method, are shown in (4.7). The contractual flow components on a line connecting buses i and j are represented by a flow allocation matrix, $P_{f_{ij}}$: where the sum of all elements of $P_{f_{ij}}$ is equal to the net flow on line i - j and $P_{f_{ij,mn}}$ represents the flow attributed to the transaction involving a generator at bus m and a load at bus n , that is, GD_{mn} . Two additional vectors, i from and i to, are defined for the purpose of interpreting the flow matrices and the vector of net line flows properly,

$$\begin{aligned} i\text{from} &= [1 \ 1 \ 2 \ 3 \ 3 \ 4] \\ i\text{to} &= [2 \ 4 \ 3 \ 4 \ 5 \ 5] \end{aligned} \quad (4.5)$$

The vector of net line flows, $P_{f_{net}}$, obtained by summing the entries of the corresponding flow matrix and verified using a full AC loadflow algorithm is,

$$P_{f_{net}} = [191.7 \quad 4.8 \quad 124.9 \quad -146.2 \quad -117.2 \quad 204.2] \quad (4.6)$$

The i^{th} entry of $P_{f_{net}}$ represents the flow in MW, injected into the line connecting buses $ifrom_i$ and ito_i , at bus $ifrom_i$. The direction of the flow component $P_{f_{ij,mn}}$ is obtained in like manner.

$$\begin{aligned}
 P_{f_{12}} &= \begin{bmatrix} 0.0 & 11.4 & 107.7 & 39.3 & 85.0 \\ -86.0 & 0.0 & -84.1 & -5.8 & -7.9 \\ -72.4 & 63.6 & 0.0 & -30.2 & -3.7 \\ -24.1 & 123.8 & 53.9 & 0.0 & 24.8 \\ -44.9 & 13.8 & 38.9 & -11.4 & 0.0 \end{bmatrix} & P_{f_{14}} &= \begin{bmatrix} 0.0 & 3.9 & 128.3 & 164.5 & 213.8 \\ -29.0 & 0.0 & 84.1 & 5.8 & 7.9 \\ -86.5 & -63.6 & 0.0 & 30.2 & 3.7 \\ -101.1 & -123.8 & -53.9 & 0.0 & -24.8 \\ -113.1 & -13.8 & -38.9 & 11.4 & 0.0 \end{bmatrix} \\
 P_{f_{23}} &= \begin{bmatrix} 0.0 & -3.6 & 99.1 & 36.3 & 78.4 \\ 26.8 & 0.0 & 188.4 & 4.3 & 8.5 \\ -67.0 & -142.5 & 0.0 & -28.0 & -3.4 \\ -22.3 & -91.6 & 49.6 & 0.0 & 22.9 \\ -41.5 & -14.8 & 35.9 & -10.5 & 0.0 \end{bmatrix} & P_{f_{34}} &= \begin{bmatrix} 0.0 & -1.7 & -57.1 & 17.6 & -8.5 \\ 13.0 & 0.0 & -37.5 & 2.1 & 1.4 \\ 38.9 & 28.5 & 0.0 & 38.1 & 4.6 \\ -10.8 & -44.4 & -67.1 & 0.0 & -31.0 \\ 4.5 & -2.5 & -48.6 & 14.4 & 0.0 \end{bmatrix} \\
 P_{f_{35}} &= \begin{bmatrix} 0.0 & -1.6 & -53.6 & 16.5 & 82.1 \\ 12.2 & 0.0 & -35.2 & 1.9 & 6.5 \\ 36.4 & 26.7 & 0.0 & 35.6 & 10.7 \\ -10.2 & -41.6 & -63.1 & 0.0 & 52.4 \\ -43.6 & -11.4 & -113.9 & -24.3 & 0.0 \end{bmatrix} & P_{f_{45}} &= \begin{bmatrix} 0.0 & 1.9 & 62.0 & -19.0 & 203.9 \\ -14.0 & 0.0 & 40.6 & -2.2 & 9.6 \\ -41.7 & -30.7 & 0.0 & -40.9 & 9.0 \\ 11.7 & 48.1 & 73.0 & 0.0 & 209.9 \\ -107.9 & -16.8 & -95.3 & -96.8 & 0.0 \end{bmatrix}
 \end{aligned} \quad (4.7)$$

The signs of the entries in the line flow allocation matrices can be interpreted in accordance with the following rule: *The flow component, $P_{f_{ij,mn}}$, and its corresponding net line flow are in the same direction if their signs are the same.* In other words, the net effect of a flow component, $P_{f_{ij,mn}}$, (in combination with all other contracts,) is to increase the net line flow whenever its sign is the same as that of its corresponding net line flow; the opposite is true when signs are opposite.

It is first noticed that all diagonal elements of the flow matrices are zero. Load contracts between generators and loads located at the same bus do not produce any line flows, since

such transactions result in zero net injection. Note that this result is consistent with the loss allocation. Since such contracts produce zero system loss regardless of system configuration, there should not be any power flow allocated to them.

Secondly, as was the case for the simultaneous and linear loss allocation, for any line, the flows allocated to pairs of contracts between two buses i and j , namely $P_{f_{ij,mm}}$ and $P_{f_{ij,nn}}$, are always of opposite sign. This is not totally unexpected since such pairs of contracts attempt to push power in opposite directions. This is not necessarily true for nonlinear integration paths.

As was the case for the loss allocation problem, it is possible to obtain reasonably accurate results using few integration steps, a property which can be exploited for the purpose of real time allocation. In addition, it is also very informative to compare the flow allocation procedure to traditional, non-physical methods such as *contract paths*, to illustrate the inadequacy of the latter. A discussion of these issues therefore follows.

4.5 Approximate and Heuristic Flow Allocation

4.5.1 Euler Approximations and the Method of Contract Paths

Alternative methods of flow allocation have been examined and compared to the exact method presented above. The first is an approximation obtained using fewer integration steps, while the second is a heuristic method, commonly referred to in literature as the method of *contract paths*.

First, approximate results based on a 2-step Euler integration are presented for two transmission lines; exact results are shown as well for convenience. As was the case for the approximate loss allocation, intermediate results were scaled as shown in (3.7) to make them loadflow consistent. Furthermore, it should be noted that contracts and corresponding contractual losses are supplied by the same generator; that is, the loss index matrix is as shown in (4.4),

$$\begin{aligned}
 P_{f_{12_{\text{int}}}} &= \begin{bmatrix} 0.0 & 11.0 & 97.7 & 38.4 & 79.9 \\ -86.4 & 0 & -79.9 & -5.8 & -7.8 \\ -71.5 & 62.8 & 0 & -30.0 & -3.5 \\ -23.6 & 120.0 & 49.2 & 0 & 23.1 \\ -43.8 & 13.5 & 36.0 & -11.1 & 0 \end{bmatrix} & P_{f_{23_{\text{int}}}} &= \begin{bmatrix} 0 & -3.5 & 94.2 & 37.1 & 77.1 \\ 27.7 & 0 & 179.0 & 4.4 & 8.4 \\ -69.1 & -140.6 & 0 & -29.0 & -3.4 \\ -22.8 & -90.1 & 47.4 & 0 & 22.3 \\ -42.3 & -14.5 & 34.7 & -10.7 & 0 \end{bmatrix} \\
 P_{f_{12}^{\text{exact}}} &= \begin{bmatrix} 0.0 & 11.4 & 107.7 & 39.3 & 85.0 \\ -86.0 & 0.0 & -84.1 & -5.8 & -7.9 \\ -72.4 & 63.6 & 0.0 & -30.2 & -3.7 \\ -24.1 & 123.8 & 53.9 & 0.0 & 24.8 \\ -44.9 & 13.8 & 38.9 & -11.4 & 0.0 \end{bmatrix} & P_{f_{23}^{\text{exact}}} &= \begin{bmatrix} 0.0 & -3.6 & 99.1 & 36.3 & 78.4 \\ 26.8 & 0.0 & 188.4 & 4.3 & 8.5 \\ -67.0 & -142.5 & 0.0 & -28.0 & -3.4 \\ -22.3 & -91.6 & 49.6 & 0.0 & 22.9 \\ -41.5 & -14.8 & 35.9 & -10.5 & 0.0 \end{bmatrix}
 \end{aligned} \tag{4.8}$$

The above results are indicative of a general trend. Reasonably accurate solutions can be obtained with as few as two integration steps. In comparison with the loss allocation procedure, fewer integration steps are required, since any errors in net line flow are distributed over a larger number of values, namely all the entries of the $P_{f_{ij}}$, thereby reducing the effect of error on any particular line flow component.

The discussion of line loading allocation would not be complete unless the method of contract paths received due consideration, as it has been the prevailing flow allocation scheme and still retains acceptance among a large portion of the power system industry. Considering the fact that it has been proven to be unacceptable for the purpose of line flow allocation, its acceptance must stem from the fact that it provides an easy method of allocating line loading to individual transactions. For the purpose of demonstrating the insufficiency of the method of *contract paths*, its solution is compared to the exact solution for three distinct bilateral transactions. These particular contracts are chosen for the simple reason that the participating generator and load buses are connected by direct link, which is chosen to define the contract path. Recall that for any line i - j , where i is less than j , the flow is positive if it is flowing from i to j , and negative if flowing from j to i . The results are given in tables (4.1), (4.2), and (4.3).

Line	Contract Path (MW)	Exact Flow (MW)
1-2	0	39.3
1-4	203.8	164.5
2-3	0	36.3
3-4	0	17.6
3-5	0	16.5
4-5	0	-19.0

Line	Contract Path (MW)	Exact Flow (MW)
1-2	0	24.8
1-4	0	-24.8
2-3	0	22.9
3-4	0	-31.0
3-5	0	52.4
4-5	253.9	209.9

Line	Contract Path (MW)	Exact Solution (MW)
1-2	0	63.6
1-4	0	-63.6
2-3	-201.3	-142.5
3-4	0	28.5
3-5	0	26.7
4-5	0	-30.7

First, it should be noted that while it is true that the main component of the flow obtained using the exact allocation procedure is on the direct link between generator and load bus, these values still differ significantly from the *contract path* flows. Furthermore, the method of contract paths neglects all other (possibly substantial) line flows. Clearly, these observations provide an adequate explanation for the presence of “loop flows” experienced by transmission providers, forcing them to curtail legitimate transactions to accommodate flows of unknown origin. The advantage of the proposed flow allocation procedure is clear, allowing transmission providers to exactly predict the impact of individual transactions on the network, thereby eliminating the possibility of unforeseen or inexplicable congestion problems. In contrast, the method of contract paths is essentially a guess, and usually not a very good one.

An additional advantage of the flow allocation scheme is that, in addition to providing the exact contractual line-flow allocations, the results can also reflect the time evolution of the load. That is, while the method of *contract paths* yields the same results regardless of the order in which contacts arrive, varying the path of integration will produce different contractual line-flow allocations. This topic is addressed in section (4.6).

4.5.2 Line Flow Approximations Using DC Loadflow

The DC loadflow formulation, although not exact, does possess practical advantages. Most importantly, the DC loadflow formulation provides a linear relationship between individual transactions and their corresponding line flow allocations. This property can be exploited by both the consumer and the ISO. The consumer, who must ultimately pay for transmission usage, can approximately predict the impact of his transaction on given line flows through knowledge of the (constant) sensitivity of each line flow with respect to the consumer's transaction. Furthermore, the ISO can use that same information for the purpose of congestion management. Unfortunately, the DC loadflow formulation may not be adequate for charging purposes, since even under moderate loading the deviation from the exact solution can be significant. As previously stated, market participants have no desire to subsidize their competitors costs; consequently, there may be no other alternative but to use the full line-loading allocation algorithm.

The results shown in (4.9) and (4.10) compare the exact solution to that obtained using the DC loadflow. Clearly, while the DC loadflow approximation provides a useful tool from both market and operational perspectives, the errors may be too large to ignore. The line flow allocation results are obtained using the transaction matrix shown in equation (4.4) and the power system shown in figure (4.1). The exact results are repeated for convenience. All results are in MW.

$$P_{f12}^{DC} = \begin{bmatrix} 0 & 10.6 & 90.8 & 37.7 & 76.2 \\ -86.7 & 0 & -76.8 & -5.9 & -7.7 \\ -70.8 & 62.0 & 0 & -30.0 & -3.5 \\ -23.1 & 117.2 & 46.1 & 0 & 22.1 \\ -43.0 & 13.3 & 34.1 & -10.9 & 0 \end{bmatrix} \quad P_{f12}^{exact} = \begin{bmatrix} 0 & 11.4 & 107.7 & 39.3 & 85.0 \\ -86.0 & 0 & -84.1 & -5.8 & -7.9 \\ -72.4 & 63.6 & 0 & -30.2 & -3.7 \\ -24.1 & 123.8 & 53.9 & 0 & 24.8 \\ -44.9 & 13.8 & 38.9 & -11.4 & 0 \end{bmatrix} \quad (4.9)$$

4.6 Path Dependent Flow Allocation

As was the case for the loss allocation, the path dependence of the flow allocation procedure was investigated by restricting the contracts to a predetermined non-linear path, defined by GDA and GDB. GDA is identical to GD with the exception that GDA_{32} is zero and all entries

$$\begin{aligned} P_{f \text{ net}}^{DC} &= [151.7 \quad -7.0 \quad 93.4 \quad -136.8 \quad -100.4 \quad 164.1] \\ P_{f \text{ net}}^{\text{exact}} &= [191.7 \quad 4.84 \quad 124.9 \quad -146.2 \quad -117.2 \quad 204.2] \end{aligned} \quad (4.10)$$

of GDB are zero except GDB_{32} which is equal to GD_{32} . This scheme represents a simplified batch arrival scheme. It is assumed that the contracts evolve linearly with time on each of the subpaths. The load matrix, GD, the loss index matrix, the flow allocation matrices and net line flows for the non-linear path are shown in equations (4.11), (4.12), and (4.13).

$$GD = \begin{bmatrix} 65.7 & 14.1 & 203.7 & 203.8 & 280.4 \\ 115.1 & 155.8 & 249.3 & 10.4 & 16.1 \\ 158.9 & 201.3 & 2.3 & 115.0 & 20.0 \\ 125.3 & 206.0 & 176.7 & 279.1 & 253.9 \\ 158.1 & 27.6 & 196.2 & 124.8 & 210.4 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (4.11)$$

The first results show the net line flows for each linear subpath, P_{FA} and P_{FB} . The linear integration path results are also repeated here for convenience.

$$\begin{aligned} P_{f \text{ net } A} &= [129.9 \quad 105.5 \quad 307.3 \quad -175.4 \quad -149.6 \quad 254.7] \\ P_{f \text{ net } B} &= [197.2 \quad 38.2 \quad 164.5 \quad -155.6 \quad -126.5 \quad 221.2] \\ P_{f \text{ net}} &= [191.7 \quad 4.9 \quad 124.9 \quad -146.2 \quad -117.2 \quad 204.2] \end{aligned} \quad (4.12)$$

Evidently, the addition of the bilateral transaction GD_{32} has a drastic affect on the net line

flows. Furthermore, there is no relation between the linear and nonlinear path results. Consequently, the path of integration cannot be arbitrary; mathematically correct results demand that the time evolution of the load matrix, $GD(t)$, be known.

The flow allocation matrices for each subpath, $P_{fA\ ij}$ and $P_{fB\ ij}$, are shown below for two of the six lines. The linear path results are also repeated here for the purpose of comparison.

$$\begin{aligned}
 P_{fA\ 12} &= \begin{bmatrix} 0 & 11.1 & 113.6 & 39.9 & 87.7 \\ -86.1 & 0 & -94.5 & -6.0 & -8.4 \\ -70.8 & 0 & 0 & -30.2 & -3.7 \\ -23.8 & 120.8 & 56.4 & 0 & 25.5 \\ -44.3 & 13.6 & 40.5 & -11.5 & 0 \end{bmatrix} & P_{fA\ 45} &= \begin{bmatrix} 0 & 1.8 & 67.3 & -18.9 & 207.5 \\ -13.8 & 0 & 45.0 & -2.2 & 9.7 \\ -41.8 & 0 & 0 & -41.8 & 8.9 \\ 11.3 & 45.4 & 78.4 & 0 & 213.4 \\ -105.1 & -15.8 & -97.8 & -96.8 & 0 \end{bmatrix} \\
 P_{fB\ 12} &= \begin{bmatrix} 0 & 11.1 & 113.6 & 39.9 & 87.7 \\ -86.1 & 0 & -94.5 & -6.0 & -8.4 \\ -70.8 & 67.2 & 0 & -30.2 & -3.7 \\ -23.8 & 120.8 & 56.4 & 0 & 25.5 \\ -44.3 & 13.6 & 40.5 & -11.5 & 0 \end{bmatrix} & P_{fB\ 45} &= \begin{bmatrix} 0 & 1.8 & 67.3 & -18.9 & 207.5 \\ -13.8 & 0 & 45.0 & -2.2 & 9.7 \\ -41.8 & -33.5 & 0 & -41.8 & 8.9 \\ 11.3 & 45.4 & 78.4 & 0 & 213.4 \\ -105.1 & -15.8 & -97.8 & -96.8 & 0 \end{bmatrix} & (4.13) \\
 P_f^{linear}\ 12 &= \begin{bmatrix} 0 & 11.4 & 107.7 & 39.3 & 85.0 \\ -86.0 & 0 & -84.1 & -5.8 & -7.9 \\ -72.4 & 63.6 & 0 & -30.2 & -3.7 \\ -24.1 & 123.8 & 53.9 & 0 & 24.8 \\ -44.9 & 13.8 & 38.9 & -11.4 & 0 \end{bmatrix} & P_f^{linear}\ 45 &= \begin{bmatrix} 0 & 1.9 & 62.0 & -19.0 & 203.9 \\ -14.0 & 0 & 40.6 & -2.2 & 9.6 \\ -41.7 & -30.7 & 0 & -40.9 & 9.0 \\ 11.7 & 48.1 & 73.0 & 0 & 209.9 \\ -107.9 & -16.8 & -95.3 & -96.8 & 0 \end{bmatrix}
 \end{aligned}$$

First, it should be observed that the only difference between $P_{fA\ 12}$ and $P_{fB\ 12}$ is the 3-2 entry. This result reflects a general rule of the flow allocation process: *Flow allocation is only changed for those contracts which have been modified*. This is consistent with equation (4.1), which states that $P_{f\ ij,mn}$ is constant if GD_{mn} is constant. As was the case for the loss allocation, it could again be argued that this is unfair, since any benefit (flow reduction) or penalty (flow increase) may be attributed to few parties, thereby neglecting the fact that all flows are a complex nonlinear function of all bilateral transactions present, regardless of

whether they have or have not been recently modified. For this reason, it is arguably more equitable to reintegrate from $t = 0$, using a linear path, thereby distributing benefit and penalties over all transactions. That is, all contracts should probably be incremented simultaneously and linearly from zero to their final values dictated by the GD matrix.

Secondly, although the final load matrix, GD, is identical, the final flow allocations are different, proving that the flow allocation algorithm is in fact path dependent.

Although it has so far been assumed that load contracts and corresponding contractual losses have been supplied by the same generator, this need not be the case in general. It is possible that a load chooses to purchase losses from a different bus for financial or technical reasons. This situation provides much insight into the nature of the flow allocation process and is therefore addressed.

4.7 Generalized Line-Loading Allocation

Purchasing contractual losses from a generator other than that supplying the load contract GD_{ij} not only affects the loss allocation, but also has a profound influence on contractual line-flow components. This behaviour is investigated using the same 5-bus example used for generalized loss allocation. The loss index matrix, U, and load matrix, GD, are repeated here for convenience,

$$GD = \begin{bmatrix} 65.7 & 14.1 & 203.7 & 203.8 & 280.4 \\ 115.1 & 155.8 & 249.3 & 10.4 & 16.1 \\ 158.9 & 201.3 & 2.3 & 115.0 & 20.0 \\ 125.3 & 206.0 & 176.7 & 279.1 & 253.9 \\ 158.1 & 27.6 & 196.2 & 124.8 & 210.4 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (4.14)$$

Recall that several of the load contracts belonging to the fourth row of GD received relatively high positive losses in the case that losses were supplied by the same generator responsible

for the load. That is, the losses corresponding to a bilateral transaction of the form, GD_{ij} , were rather large when supplied by generator i . It was shown that the magnitude of the contractual losses under consideration could be substantially reduced by having the corresponding losses supplied by the generator located at the load bus, the reason being that injecting positive losses at the load bus balanced the distribution of net injections across the system, and in particular, decreased the difference between the bus injections corresponding to the relevant bilateral transactions. A similar effect can also be observed regarding the magnitude of the contractual line-flow components. The net line flow vector, $P_{f_{net}}$, and the flow matrices, $P_{f_{ij}}$, are shown below. All results are in MW.

$$P_{f_{net}} = [172.0 \quad 22.5 \quad 122.4 \quad -133.7 \quad -108.3 \quad 176.7] \quad (4.15)$$

$$\begin{aligned}
 P_{f_{12}} &= \begin{bmatrix} 0.0 & 11.4 & 106.5 & 39.4 & 84.5 \\ -86.0 & 0.0 & -84.0 & -5.8 & -7.9 \\ -72.2 & 63.6 & 0.0 & -30.2 & -3.6 \\ -24.2 & 114.8 & 46.4 & 0.0 & 23.1 \\ -44.8 & 13.8 & 38.6 & -11.4 & 0.0 \end{bmatrix} & P_{f_{14}} &= \begin{bmatrix} 0.0 & 3.8 & 127.8 & 165.2 & 213.7 \\ -29.0 & 0.0 & 84.0 & 5.8 & 7.9 \\ -86.7 & -63.6 & 0.0 & 30.2 & 3.6 \\ -101.5 & -114.8 & -46.4 & 0.0 & -23.1 \\ -113.3 & -13.8 & -38.6 & 11.4 & 0.0 \end{bmatrix} \\
 P_{f_{23}} &= \begin{bmatrix} 0.0 & -3.6 & 98.7 & 36.6 & 78.4 \\ 26.9 & 0.0 & 187.8 & 4.3 & 8.5 \\ -67.1 & -142.2 & 0.0 & -28.1 & -3.4 \\ -22.5 & -85.4 & 43.2 & 0.0 & 21.5 \\ -41.6 & -14.8 & 35.9 & -10.6 & 0.0 \end{bmatrix} & P_{f_{34}} &= \begin{bmatrix} 0.0 & -1.7 & -57.6 & 17.8 & -8.5 \\ 13.2 & 0.0 & -37.9 & 2.1 & 1.5 \\ 39.4 & 28.8 & 0.0 & 38.7 & 4.7 \\ -11.0 & -41.7 & -59.4 & 0.0 & -29.5 \\ 4.5 & -2.5 & -49.0 & 14.5 & 0.0 \end{bmatrix} \\
 P_{f_{35}} &= \begin{bmatrix} 0.0 & -1.6 & -53.5 & 16.6 & 82.2 \\ 12.2 & 0.0 & -35.2 & 1.9 & 6.5 \\ 36.6 & 26.8 & 0.0 & 35.9 & 10.8 \\ -10.2 & -38.7 & -55.1 & 0.0 & 49.8 \\ -43.7 & -11.3 & -113.6 & -24.5 & 0.0 \end{bmatrix} & P_{f_{45}} &= \begin{bmatrix} 0.0 & 1.8 & 61.2 & -18.9 & 203.0 \\ -13.9 & 0.0 & 40.2 & -2.2 & 9.6 \\ -41.5 & -30.4 & 0.0 & -40.8 & 8.9 \\ 11.6 & 44.1 & 62.7 & 0.0 & 197.1 \\ -107.7 & -16.6 & -94.6 & -96.9 & 0.0 \end{bmatrix}
 \end{aligned} \quad (4.16)$$

Comparing the allocations shown in equations (4.16) and (4.7) indicates that the significant changes in the line-flow allocations occur in the fourth row of each flow allocation matrix. That is, the contractual flows allocated to bilateral transactions for which the loss index matrix entry has not changed are relatively unmodified. Any slight variation in these allocations can be easily understood by considering that modifying any loss index matrix entry results in a redistribution of generation, thereby modifying the net injections. Consequently, the relative relationships between all buses has been slightly perturbed. Nonetheless, these effects are negligible in comparison to the changes observed for contractual flows of the form $P_{f_{ij},4n}$.

It is important to note that reducing positive losses as shown in chapter 3, and thereby modifying line-flow components, using the generalized allocation procedure, can be accomplished using a variety of generators for the purpose of supplying losses. For example, similar results could have been obtained by using the loss index matrix,

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (4.17)$$

As mentioned in the discussion of generalized loss allocation in the previous chapter, the most important issue is not whether any particular bus is a net load or generation bus, but rather the relative relationship between buses.

Security is always of prime importance in power system operation. Knowledge of the flow sensitivity matrices provided by the flow allocation algorithm enable the ISO to manage congestion. In particular, the ISO can determine exactly which load contracts will alleviate line loading. This issue is now addressed.

4.8 Flow Allocation, Congestion Management, and Choice of Supplier

In a deregulated environment it is very probable that a set of proposed transactions results in a violation of one or more security constraints, namely the line flow limits. Providing market participants with sensitivity factors does not entirely alleviate the problem, since they do not have prior knowledge of all other concurrent transactions. Since different load contracts can have opposite affects on a line flow, that is, one contract may increase the net flow while another may decrease it, sensitivity factors may mislead buyers and sellers to believe that a feasible contract is infeasible. Consequently, it is proposed that, from the perspective of the market, demanding that security issues be addressed at the market level in the decision making process, as is suggested in [12], is not necessarily the most efficient or economical way to ensure system security.

For the purpose of demonstrating the use of the flow allocation algorithm in congestion management it is assumed that a secure operating point, defined by GD , L , and U has been achieved. GD , L , and U are chosen to be the same matrices used earlier for loss and line flow allocation for the linear integration path. Recall that load contracts and contractual losses are generated at the same bus. The generation and load matrices, as well as the net line flow vector, are repeated below for convenience,

$$GD = \begin{bmatrix} 65.7 & 14.1 & 203.7 & 203.8 & 280.4 \\ 115.1 & 155.8 & 249.3 & 10.4 & 16.1 \\ 158.9 & 201.3 & 2.3 & 115.0 & 20.0 \\ 125.3 & 206.0 & 176.7 & 279.1 & 253.9 \\ 158.1 & 27.6 & 196.2 & 124.8 & 210.4 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad (4.18)$$

$$P_{f_{net}} = [191.7 \quad 4.9 \quad 124.9 \quad -146.2 \quad -117.2 \quad 204.2] \quad (4.19)$$

Suppose that lines 1-2 and 4-5, indicated by the first and last entries of $P_{f_{net}}$ respectively, are heavily loaded and operating close to their transfer limits. It is possible to determine which transactions will help relieve this congestion by inspecting the line flow sensitivity matrices, H_f , given below, corresponding to each of these lines,

$$\begin{aligned}
 H_{f_{12}} &= \begin{bmatrix} 0 & 0.88 & 0.65 & 0.20 & 0.34 \\ -0.74 & 0 & -0.37 & -0.54 & -0.51 \\ -0.47 & 0.32 & 0 & -0.27 & -0.20 \\ -0.20 & 0.65 & 0.37 & 0 & 0.11 \\ -0.30 & 0.53 & 0.24 & -0.10 & 0 \end{bmatrix} \\
 H_{f_{45}} &= \begin{bmatrix} 0 & 0.16 & 0.37 & -0.10 & 0.77 \\ -0.13 & 0 & 0.19 & -0.23 & 0.62 \\ -0.27 & -0.16 & 0 & -0.37 & 0.46 \\ 0.10 & 0.28 & 0.51 & 0 & 0.89 \\ -0.67 & -0.64 & -0.55 & -0.77 & 0 \end{bmatrix}
 \end{aligned} \tag{4.20}$$

Recall that a transaction GD_{ij} will increase a net line flow if the sign of ij^{th} entry of the corresponding matrix, H_f , is of the same sign as the net line flow. Furthermore, observe that a transaction which reduces the flow on one congested line may worsen the situation on another. For example, $H_{f_{12,23}}$ is -0.37, that is, its sign is opposite that of the net line flow, indicating that a transaction between a generator at bus 2 and a load at bus 3 will relieve congestion on line 1-2. In contrast, the same transaction will increase the flow on line 4-5. Hence, although not always possible, it is desirable to find a transaction which will reduce both line flows. For this example, the transaction, GD_{31} will accomplish this goal.

The expected change in net line flows can be approximated using differential calculus. Recall that each line-flow is described by a corresponding matrix differential equation of the form,

$$\frac{dP_f}{dt} = H_f(t) \cdot \frac{dGD}{dt} \quad (4.21)$$

so that the approximation,

$$\Delta P_f = H_f(t) \cdot \Delta GD \quad (4.22)$$

is valid for small increments in the load matrix GD. The differential equation corresponding to the line-flows and transaction being considered, namely lines 1-2 and 4-5 and contract GD_{51} , are obtained by extracting the corresponding entries of the relevant matrix equations described by (4.21),

$$\begin{aligned} \Delta P_{f_{1-2}} &= H_{f_{12,51}} \cdot \Delta GD_{51} \\ \Delta P_{f_{4-5}} &= H_{f_{45,51}} \cdot \Delta GD_{51} \end{aligned} \quad (4.23)$$

Consequently, for $\Delta GD_{51} = 40 MW$, the updated line flows could be approximated to be $P_{F1-2} = 179 MW$ and $P_{F4-5} = 177 MW$. These approximations are validated by the exact updated MW line flows, P_f^* shown below.

$$P_{f_{net}}^* = [179.8 \quad -23.3 \quad 114.9 \quad -145.2 \quad -126.9 \quad 177.2] \quad (4.24)$$

Therefore, if it is known that a load located at bus 1 wishes to purchase power, an incentive can be given for obtaining the power from the generator at bus 5 in order to relieve congestion. The above results also indicate that a market participant can exercise some degree of control over his line flow allocation through knowledge of the line flow sensitivity matrices, allowing him to predict the approximate impact of a proposed transaction on the

system flows.

4.9 Summary and Conclusions

The results presented in this chapter are based on the notion that an infinitesimal increment in a bus-to-bus transaction at the receiving end affects only the value of its corresponding contractual line flow components. Furthermore, for an infinitesimal variation in a load contract, it is possible to exactly determine the corresponding real line flow allocation. Numerical examples illustrate the theory.

Briefly, experimental results indicate:

- (1) The line-loading allocation problem is path dependent.
- (2) Reintegrating all contracts from $t = 0$ is arguably more equitable, since it distributes any penalty or benefit resulting from the introduction of any new transactions.
- (3) Approximate and reasonable accurate allocations can be obtained using relatively few integration steps.
- (4) The method of contract paths does not reflect the actual line-flow and is therefore insufficient for allocation purposes.
- (5) The line flow sensitivity matrices and DC loadflow formulations can be used for the purpose of congestion management, thereby aiding a PSO maintain security and reliability.

The analysis and discussion presented in this chapter is not exhaustive. Further research is required to better understand the allocation procedure, and in particular, the interaction between the various transactions.

Chapter 5

30 Bus Power System

5.1 Introduction

The previous chapters dealt with the analysis and discussion of the fundamental properties of the proposed allocation procedure. Results were shown for two and five bus power systems. The purpose of the present chapter is to present, for completeness, results corresponding to a larger, thirty bus system. For convenience, the flow allocations are discussed for only two lines and all allocation results are presented for a 4x4 sub-block. The system parameters can be found in appendix A.

5.2 Notation

5.2.1 Transactions Matrix

The entire matrix of transactions, GD , can be found in appendix A.1. For simplicity, only a 4x4 block of transactions described by \overline{GD} will be considered for purpose of discussion.

$$\overline{GD} = \begin{bmatrix} GD_{12} & GD_{15} & GD_{18} & GD_{1\ 19} \\ GD_{22} & GD_{25} & GD_{28} & GD_{2\ 19} \\ GD_{52} & GD_{55} & GD_{58} & GD_{5\ 19} \\ GD_{82} & GD_{85} & GD_{88} & GD_{8\ 19} \end{bmatrix} \quad (5.1)$$

All subsequent results correspond to the transaction matrix in equation (5.1). The numerical MW transaction matrix is shown in equation (5.2).

$$\overline{GD} = \begin{bmatrix} 2.8 & 6.9 & 0.1 & 3.0 \\ 8.3 & 6.8 & 0.6 & 7.4 \\ 6.1 & 5.5 & 3.6 & 7.6 \\ 5.5 & 6.0 & 0.8 & 7.7 \end{bmatrix} \quad (5.2)$$

Two cases are considered. First, it is assumed that each consumer purchases both its load contract and losses from the same generator, whereas, in the second case, all contractual losses are purchased from a generator located at bus 24, the purpose of the latter case being to address measures which can substantially reduce contractual or total losses.

5.2.2 Conventions and Interpretations

The conventions for interpretation of any data are as before. First of all, a positive loss allocation implies that a transaction tends to increase the total system losses, while a negative loss allocation indicates that the net effect of a transaction is to decrease the total system losses. The interpretation of the line flow allocation results are only slightly more complicated. First, it must be noted that all transmission lines are designated by an origin bus, designated by the “from” column of table (A.1), and a destination bus, as shown in the “to” column of table (A.1). A line flow is referenced with respect to the origin bus and is defined to be a positive flow if directed from the origin bus to the destination bus. More detailed examples using these conventions can be found in chapters 3 and 4.

It is important to note that negative or positive allocations are the result of incremental accumulations occurring at each numerical integration step. As previously explained in chapters 3 and 4, the sign of incremental losses or line flows remain unchanged throughout an integration procedure during which all transactions evolve linearly and simultaneously in time. That is, the initial trend continues throughout the entire integration process.

5.2.3 Numerical Solution of Allocation Problem

The numerical results are obtained by solving the system of differential equations,

$$\frac{dL}{dt} = H_L(t) \cdot \frac{dGD}{dt} \quad (5.3)$$

where $H_{Lij} = \frac{\alpha_j - \alpha_i}{\alpha_k}$, and

$$\frac{dP_f}{dt} = H_f \cdot \frac{dGD}{dt} \quad (5.4)$$

where $H_{fij} = \beta_i - \beta_j + \beta_k \left(\frac{\alpha_j - \alpha_i}{\alpha_k} \right)$, subject to the relation,

$$\alpha^T \left[\frac{\partial P(\delta)}{\partial \delta} \right] = 0 \quad (5.5)$$

and the loadflow equation,

$$P_g - P_d = (GD - GD^T) e + \bar{L} = P(\delta) \quad (5.6)$$

where \bar{L}_i represents the amount of generation at bus i dedicated to supplying contractual losses, and β is defined by

$$\beta^T = \left[\frac{\partial P_{flow}(\delta)}{\partial \delta} \right]^T \left[\frac{\partial P(\delta)}{\partial \delta} \right]^{-1} \quad (2.27)$$

$P(\delta)$ is the vector function relating net real power injections to voltage angles, P_{flow} is an arbitrary net line flow with a corresponding matrix of line flow components, P_f . Recall, as discussed in section (2.5), the sum of the entries of P_f equal P_{flow} .

5.3 Loss and Flow Allocation

5.3.1 Load and Contractual Losses From Same Generator

All results are in MW. Table (5.1) provides the final net bus generation, load, and injection values. Inconsistencies are the result of truncation errors.

Bus	Pg	Pd	P
1	83.7	0	83.7
2	114.8	29.7	85.1
3	0	34.9	-34.9
4	0	20.2	-20.2
5	119.0	32.5	86.5
6	0	0	0
7	0	22.9	-22.9
8	109.8	15.1	94.7
9	0	0	0
10	0	37.9	-37.9
11	105.3	0	105.3
12	0	38.0	-38.0
13	0	23.2	-23.2
14	0	21.1	-21.1
15	104.7	26.9	77.8

16	0	24.1	-24.1
17	0	21.3	-21.3
18	0	32.2	-32.2
19	0	34.7	-34.7
20	0	26.1	-26.1
21	0	25.5	-25.5
22	0	0	0
23	0	16.1	-16.1
24	0	39.7	-39.7
25	0	0	0
26	0	34.8	-34.8
27	0	0	0
28	0	0	0
29	0	21.7	-21.7
30	0	24.0	-24.0

$$L = 10^{-3} \begin{bmatrix} 6.9 & -59.9 & 29.3 & 328.3 \\ 0 & -75.6 & 16.0 & 788.8 \\ 68.8 & 0 & 138.2 & 908.3 \\ -141.2 & -218.9 & 0 & 593.7 \end{bmatrix} \quad (5.7)$$

$$\begin{aligned}
 P_{f1} &= 10^{-1} \begin{bmatrix} 23.5 & 51.3 & 0.7 & 21.3 \\ 0 & -5.8 & -1.1 & -16.2 \\ 5.3 & 0 & -3.4 & -9.4 \\ 9.8 & 5.4 & 0 & -1.7 \end{bmatrix} \\
 P_{f4} &= 10^{-1} \begin{bmatrix} 4.5 & 16.6 & 0.3 & 11.7 \\ 0 & 5.7 & 1.1 & 15.7 \\ -5.1 & 0 & 3.3 & 9.1 \\ -9.5 & -5.3 & 0 & 1.6 \end{bmatrix}
 \end{aligned} \tag{5.8}$$

First, regarding the loss matrix, it should first be noted that some transactions are allocated positive losses while some are attributed negative losses. For example, transaction GD_{12} is allocated contractual losses of 6.9 kilowatts, implying that the net effect of this transaction (in combination with all other transactions) is to increase the total system losses. In contrast, transaction GD_{15} is allocated -59.9 kilowatts, indicating that its net effect (in combination with all other transactions) is to decrease the total system losses. Consequently, the total generation required to supply the load of 6.9 MW defined by GD_{15} is 6.84 MW, that is, $6.9 - 0.0599 \text{ MW}$.

Recall that it was previously stated in chapter 3 that the contractual losses corresponding to a transaction between a generator at bus i and a load at bus j are generally positive when $P_i - P_j \gg 0$ and negative when $P_i - P_j \ll 0$. Contractual losses do not necessarily adhere to this rule when the corresponding difference in net bus injections is near zero, as is the case for transactions involving two net generation buses such as those between buses 1,2,5, and 8. The sign of the contractual losses is difficult to predict under such circumstances. In contrast, the results of transactions involving a net generation bus and a net load bus, such as those indicated by the last column of \overline{GD} , are easier to predict. In accordance with the stated rule, the contractual losses corresponding to these transactions are positive. It is also important to realize that the contractual loss allocations corresponding to pairs of transactions

involving two buses, such as GD_{25} and GD_{52} or GD_{58} and GD_{85} , are of opposite sign. The reason for this is that such transactions attempt to push power in opposite directions. Lastly, it is remarked that transactions involving a generator and a load located at the same bus, such as GD_{22} , GD_{55} , and GD_{88} , are allocated zero loss, since such transactions result in zero net injection and therefore have no impact on the network.

An explanation of contractual line flow allocation is extremely difficult, since the distribution of a single line flow over all transactions can not in general be addressed independently from all other line flows. Nonetheless, some observations regarding the fundamental characteristics of the flow allocation procedure can be made. First, as in loss allocation, the contractual flows allocated to pairs of contracts between two buses, such as GD_{25} and GD_{52} or GD_{58} and GD_{85} , are always of opposite sign, since they produce opposing network flow components. Secondly, transactions between a generator and a load located at the same bus are attributed zero flow, since they have no impact on the network. Unfortunately, a qualitative understanding of the flow allocation is rather difficult, the reason being that the direction of a contractual line flow component cannot in general be predicted for a complex network.

5.3.2 All Contractual Losses Supplied By Generator Twenty-Four

All results are in MW. Table 5.2 provides the net bus generations, loads, and injections. Slight inconsistencies are the result of truncation errors.

Bus	Pg	Pd	P
1	78.1	0	78.1
2	106.8	29.7	77.1
3	0	34.9	-34.9
4	0	20.2	-20.2

5	111.7	32.5	79.2
6	0	0	0
7	0	22.9	-22.9
8	104.5	15.1	89.4
9	0	0	0
10	0	37.9	-37.9
11	100.9	0	100.9
12	0	38.0	-38.0
13	0	23.2	-23.2
14	0	21.1	-21.1
15	100.6	26.9	73.7
16	0	24.1	-24.1
17	0	21.3	-21.3
18	0	32.2	-32.2
19	0	34.7	-34.7
20	0	26.1	-26.1
21	0	25.5	-25.5
22	0	0	0
23	0	16.1	-16.1
24	29.9	39.7	-9.8
25	0	0	0
26	0	34.8	-34.8
27	0	0	0
28	0	0	0

29	0	21.7	-21.7
30	0	24.0	-24.0

$$L = 10^{-3} \begin{bmatrix} 6.1 & -50.7 & 2.4 & 286.8 \\ 0 & -64.7 & 13.4 & 691.5 \\ 58.0 & 0 & 114.6 & 782.5 \\ -122.8 & -191.0 & 0 & 547.6 \end{bmatrix} \quad (5.9)$$

$$P_{f1} = 10^{-1} \begin{bmatrix} 23.4 & 51.7 & 0.7 & 19.2 \\ 0 & -6.0 & -1.1 & -14.7 \\ 5.4 & 0 & -3.2 & -8.4 \\ 9.8 & 5.4 & 0 & -1.6 \end{bmatrix} \quad (5.10)$$

$$P_{f4} = 10^{-1} \begin{bmatrix} 4.4 & 16.8 & 0.3 & 10.6 \\ 0 & 5.8 & 1.0 & 14.3 \\ -5.2 & 0 & 3.2 & 8.2 \\ -9.5 & -5.3 & 0 & 1.5 \end{bmatrix}$$

It should be noted that there are again both positive and negative loss and line flow allocations. Regarding losses, the most significant result of having losses supplied by generator 24 is that the magnitude of most contractual losses have been reduced. More specifically, both penalties (positive losses) and rewards (negative losses) have been reduced. For example, inspection of the last column of the MW loss matrix reveals a significant decrease in contractual losses. Qualitatively, the reduction in positive losses is related to the difference in net power injections between the buses involved in the transaction. To

understand this result, it must first be noted that the last column of the loss matrix corresponds to transactions involving a transfer of power to bus 19 which happens to be a net load bus. For each of these transactions involving a generator i and load 19, $P_i - P_{19} \gg 0$. Hence, contractual losses are positive. In the previous section, these contractual losses were supplied by the same generator responsible for the load contract resulting in an even greater difference in corresponding net bus injections. Having contractual losses supplied by generator 24 introduces another factor, that is the difference between generator 24 and load 19, $P_{24} - P_{19}$. If the total aggregate transaction is considered to be the sum of the load and contractual losses, this latter scenario is equivalent to having part of the transaction supplied by generator 24. Bus i is therefore responsible for a smaller generation component and therefore produces less positive losses. Furthermore, since $P_{24} - P_{19} \ll 0$, generator 24 contributes a negative loss component. The combined effect of these factors results in smaller contractual losses.

The reduced magnitude of the negative loss allocations have a similar explanation. Unfortunately, the analysis of the present example is slightly confusing as a result of the fact that the difference in bus injections corresponding to the transactions having negative loss allocations are near zero. In fact, only one of these transactions, GD_{15} , between a generator i and load j satisfies the relation $P_i - P_j < 0$. Consequently, the contractual loss allocation do not adhere to the general rules outlined in chapter 3. Nonetheless, a qualitative understanding of the fundamental principles is still possible.

For example, consider the transaction between generator 8 and load 5, GD_{85} . When supplied by generator 8, its contractual losses were -0.219 MW, whereas they increased to -0.191 MW when supplied by generator 24. The reason for the decrease in benefit occurring when losses are supplied by bus 24 can be understood by realizing that the addition of a negative loss component at bus 24 is tantamount to introducing an additional load which must in effect be supplied by all other generators. Since $P_i - P_{24} \gg 0$ for any generator i , the addition of a load at bus 24 results in positive loss components. This consequently reduces the net benefit

experienced by transaction GD_{gs} .

5.4 Concluding Remarks

The qualitative explanations of positive and negative allocation results serve two purposes:

- (1) Develop an intuitive understanding of the impact of bilateral transactions upon the network.
- (2) Justify the nature of the results, negative allocations in particular, and relate them to network operating parameters.

It is imperative to remark that an intuitive understanding of the allocation problem, although perhaps useful, is not sufficient for operational purposes. It is highly recommended that decisions be made based on actual network parameters. More specifically, the formulation of bilateral transactions between suppliers and consumers, as well as the choice of loss supplier, should be determined using the vector α defined by the relation $\alpha^T \frac{\partial P}{\partial \delta} = 0$, and the flow sensitivity vector, β , defined in equation (2.27). The vectors α and β completely determine the exact incremental variation in system losses and line flows resulting from incremental variations in bilateral transactions between generators and loads. Although they do not enable a consumer to predict the exact loss or line flow allocation corresponding to a large change in a load contract, they can aid the consumer make beneficial economic decisions.

Chapter 6

Summary and Conclusions

6.1 Introduction

This thesis has presented a general theory for allocating contractual losses and line flows to individual bilateral transactions. The purpose of the proposed allocation theory is to provide a sound, mathematical framework for calculating the effect of individual bilateral transactions on a power network in a competitive environment. The developed theory is founded on the notion that it is always possible to determine the exact variation in contractual losses and line flows corresponding to an infinitesimal increment in a load contract. The exact loss and line flow allocation solution is then shown to be governed by a combined set of algebraic and nonlinear differential equations, whose solution is addressed using several numerical examples illustrating the fundamental properties of the allocation procedure.

6.2 Summary of Loss Allocation Results

Briefly, the loss allocation results indicate:

- (1) Negative contractual loss allocation implies that the net effect of the corresponding transaction (in combination with all the other transactions) is to reduce total system losses.
- (2) The loss allocation procedure is path dependent.
- (3) Reintegrating all contracts from $t = 0$ may be more equitable, as it distributes any penalty or benefit that may result from the introduction of any new transactions. It is also important to note that reintegrating from $t = 0$ modifies

all transactions including those which have remained constant. That is, the complex interaction between all transactions causes all loss allocations to be varied.

(4) It is possible to obtain an approximate and adequate loss allocation using relatively few integration steps, allowing participants to calculate a simple and rapid approximate allocation solution.

(5) Heuristic allocation schemes are inaccurate since they cannot possibly reflect the complex nonlinear relationships among transactions.

(6) The loss allocation algorithm can be used in the dispatching process or for the purpose of distributing loss in the case that loss replacement is solely an ancillary service. That is, the allocation solution can be used to determine the exact amount of generation to dispatch for the purpose of supplying the load demand defined by the transaction matrix, GD.

(7) There are measures that consumers can take to minimize penalty (positive loss) or maximize benefit (negative loss), the most important consideration being that a participant should attempt to purchase power from load buses. The sign and magnitude of contractual losses corresponding to a bilateral transaction between generator i and load j is dependent upon the difference, $P_i - P_j$. If the difference is much greater than zero, losses are positive, while they are negative when the difference is considerably less than zero.

6.3 Summary of Flow Allocation Results

Briefly, experimental results indicate:

(1) Negative contractual line flow allocations imply that the net effect of a transaction is to produce line flow components which oppose the direction of the

net line flow. It is important to note that a transaction can increase the flow on one line while decreasing the flow on another.

(2) The line-loading allocation problem is path dependent.

(3) Reintegrating all contracts from $t = 0$ is arguably more equitable, since it distributes any penalty or benefit resulting from the introduction of any new transactions. Nonetheless, it must be noted that reintegrating alters the flow allocation corresponding to transactions that have remained constant, an occurrence which can also be viewed as unfair.

(4) Approximate and reasonable accurate allocations can be obtained using as few as 2 integration steps.

(5) The method of contract paths does not reflect the actual line-flow and is therefore inadequate for allocation purposes.

(6) The exact line flow sensitivity matrices can be used for the purpose of congestion management, thereby aiding a PSO maintain security and reliability. The line flow sensitivity matrices are a function of the operating point and are therefore the most appropriate tool for estimating the impact of an additional contract on the network flows. The constant line flow sensitivities obtained from a DC load flow formulation can also be used, but the corresponding results can be shown to deviate significantly from those obtained using the exact sensitivities. The accuracy of the DC load flow formulation is highly dependent on the validity of the corresponding assumptions.

6.4 Future Research

The set of nonlinear differential equations governing the loss and flow allocation solution has been solved using an Euler numerical integration procedure. For the purpose of decreasing computation time, it is recommended that the allocation problem be solved using faster, more robust numerical integration procedures such as fourth or fifth order Runge-Kutta, predictor-corrector methods, or more modern multi-step methods.

Although not presently known, it is also possible that valid, equitable approximation methods based on the allocation theory can be developed in the future. Further study is recommended to provide insight into the relationship between sign and magnitude of contractual loss and flow allocations and the state of the network as defined by net bus injections. Intuitive understanding of the allocation procedure is always appreciated and would be of great benefit to marketers and consumers.

Lastly, it is recommended that the loss allocation problem be solved for non-flat voltage profiles and compared to the results contained in this thesis.

Appendix

A.1 Power System Data

Line #	From	To	r	x
1	1	2	0.0192	0.0575
2	1	3	0.0452	0.1852
3	2	4	0.0570	0.1737
4	3	4	0.0132	0.0379
5	2	5	0.0472	0.1983
6	2	6	0.0581	0.1763
7	4	6	0.0119	0.0414
8	5	7	0.0460	0.1160
9	6	7	0.0267	0.0820
10	6	8	0.0120	0.0420
11	6	9	0.0000	0.2080
12	6	10	0.0000	0.5560
13	9	11	0.0000	0.2080
14	9	10	0.0000	0.1100
15	4	12	0.0000	0.2560
16	12	13	0.0000	0.1400
17	12	14	0.1231	0.2559
18	12	15	0.0662	0.1304
19	12	16	0.0945	0.1987
20	14	15	0.2210	0.1997
21	16	17	0.0824	0.1923
22	15	18	0.1070	0.2185
23	18	19	0.0639	0.1292
24	19	20	0.0340	0.0680
25	10	20	0.0936	0.2090
26	10	17	0.0324	0.0845
27	10	21	0.0348	0.0749
28	10	22	0.0727	0.1499
29	21	22	0.0116	0.0236
30	15	23	0.1000	0.2020
31	22	24	0.1150	0.1790
32	23	24	0.1320	0.2700
33	24	25	0.1885	0.3292
34	25	26	0.2544	0.3800
35	25	27	0.1093	0.2087

Appendix

36	28	27	0.0000	0.3960
37	27	29	0.2198	0.4153
38	27	30	0.3202	0.6027
39	29	30	0.2399	0.4533
40	8	28	0.0636	0.2000
41	6	28	0.0169	0.059

A.2 Transaction Matrix

$$\begin{aligned}
 \text{genind} &= [1 \ 2 \ 5 \ 8 \ 11 \ 15 \ 24] \\
 \text{ldind} &= [2 \ 3 \ 4 \ 5 \ 7 \ 8 \ 10 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 23 \ 24 \ 26 \ 29 \ 30]
 \end{aligned} \tag{A.1}$$

The transaction matrix entries can be interpreted using the generator index and load index vectors provided in equation (A.1). The i - j entry of the transaction matrix corresponds to the contract between the generator given by the i^{th} entry of the generator index vector and the j^{th} entry of the load index vector. This notation eliminates the need to display all the zero transactions involving bus having no generators.

$$GD = \begin{bmatrix} 28 & 82 & 37 & 69 & 31 & 1 & 10 & 52 & 78 & 22 & 6 & 7 & 16 & 72 & 30 & 5 & 48 & 46 & 52 & 45 & 15 & 29 \\ 83 & 70 & 58 & 68 & 33 & 6 & 99 & 54 & 0 & 86 & 3 & 38 & 49 & 58 & 74 & 68 & 54 & 4 & 12 & 97 & 19 & 35 \\ 61 & 61 & 13 & 55 & 95 & 36 & 84 & 97 & 26 & 25 & 42 & 81 & 90 & 24 & 76 & 69 & 45 & 26 & 67 & 12 & 21 & 11 \\ 55 & 1 & 18 & 60 & 43 & 8 & 69 & 38 & 84 & 12 & 97 & 3 & 8 & 49 & 77 & 93 & 25 & 36 & 77 & 50 & 75 & 67 \\ 68 & 76 & 4 & 23 & 22 & 56 & 65 & 62 & 37 & 22 & 23 & 65 & 17 & 74 & 16 & 5 & 66 & 12 & 95 & 48 & 62 & 91 \\ 2 & 59 & 72 & 50 & 5 & 44 & 52 & 77 & 7 & 44 & 98 & 47 & 33 & 45 & 74 & 21 & 17 & 37 & 94 & 96 & 25 & 7 \\ 0 & 0 \end{bmatrix} \text{ kW} \tag{A.2}$$

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